Lensing Emulator

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Accurate cosmological emulator for the probability distribution function of gravitational lensing of point sources

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ACE_LENSING

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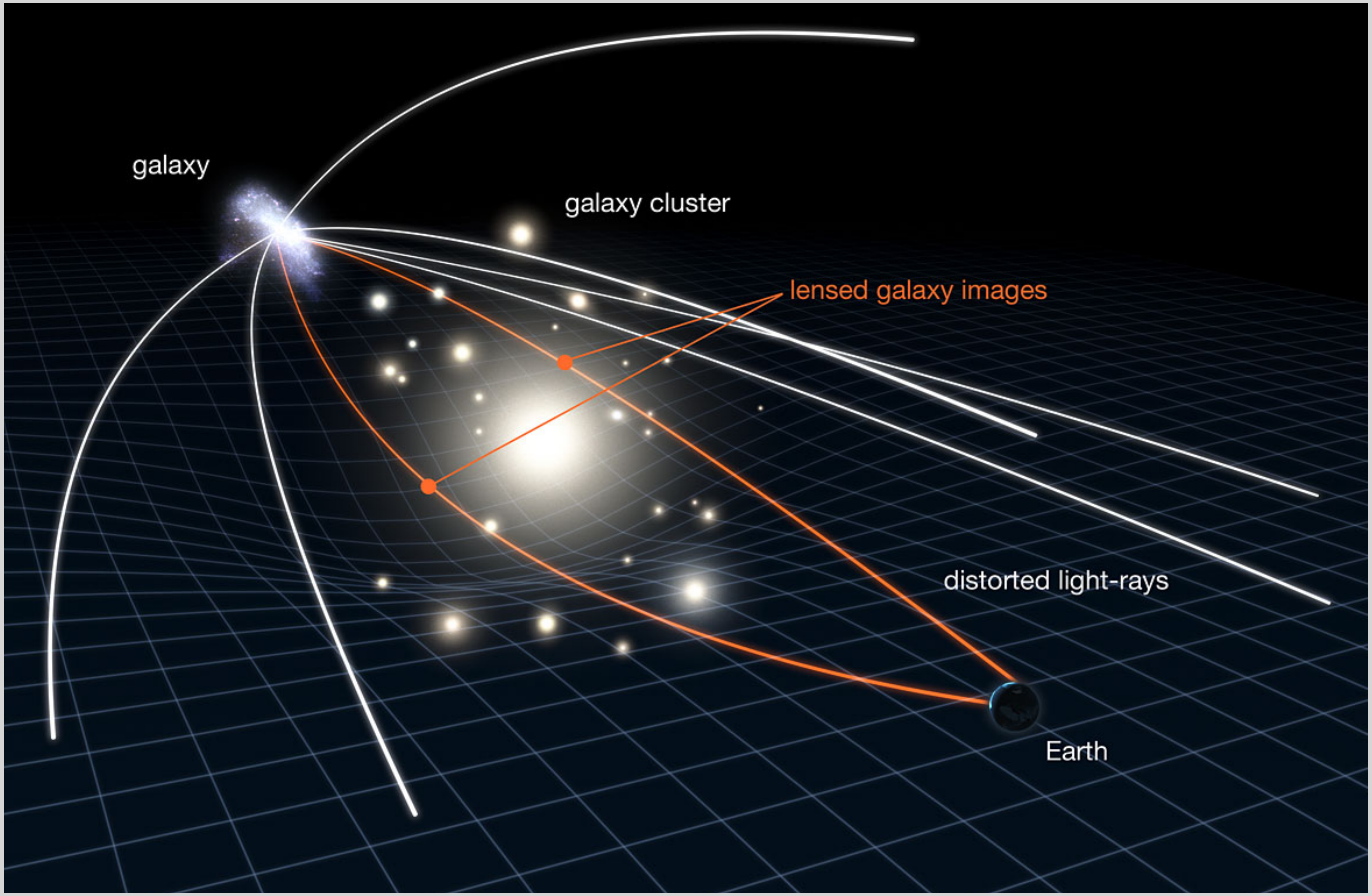
Gravitational Lensing



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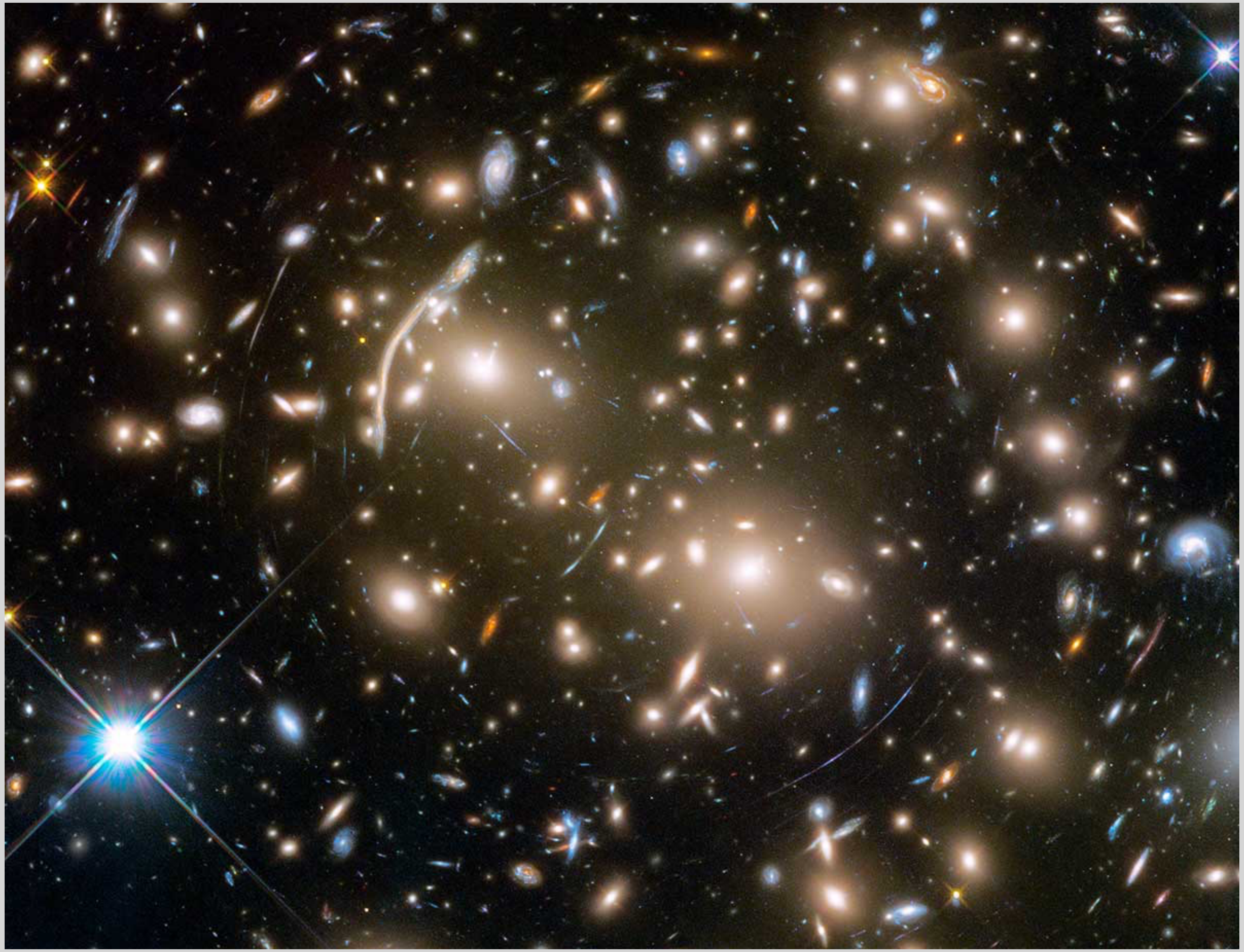
Gravitational Lensing



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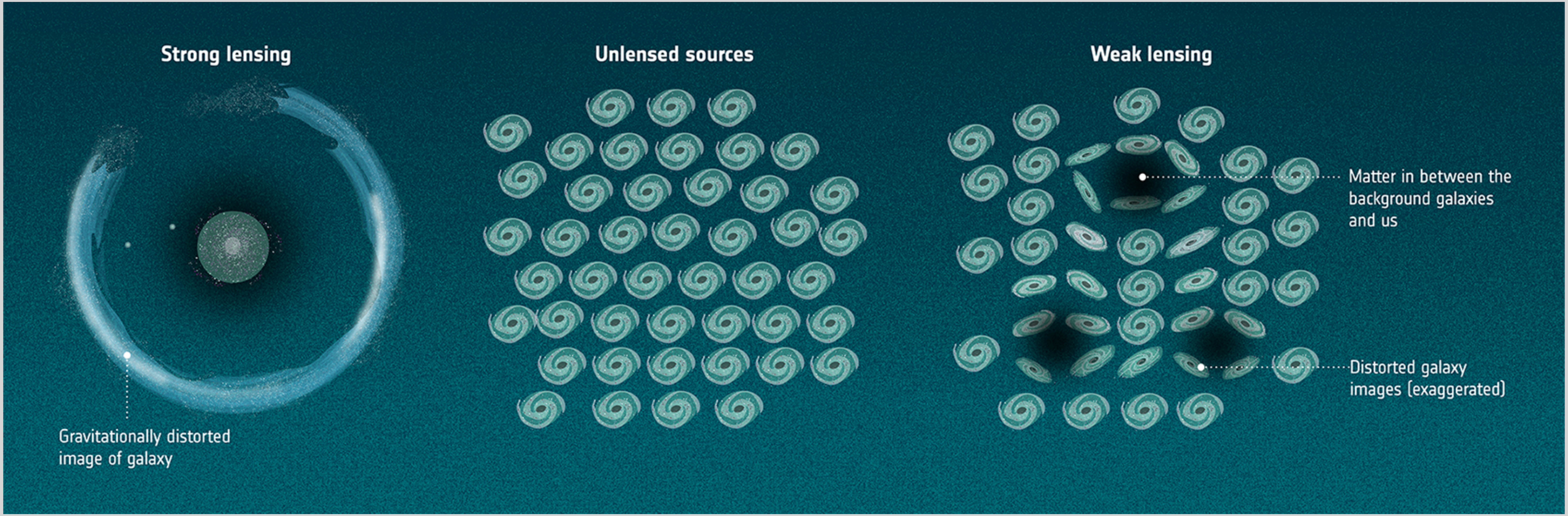
Gravitational Lensing



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Gravitational Lensing

γ as moving with velocity:

$$\mathbf{v} \approx c\,\hat{\ell}$$

The force is

$$\mathbf{F} \propto -\nabla\Phi$$

Decompose:

$$\nabla\Phi = (\nabla_\perp\Phi) + (\partial_\ell\Phi)\hat{\ell}$$

Gravitational Lensing

γ as moving with velocity:

$$\mathbf{v} \approx c\,\hat{\ell}$$

The force is

$$\mathbf{F} \propto -\nabla\Phi$$

Decompose:

changes direction

changes speed

$$\nabla\Phi = (\nabla_\perp\Phi) + (\partial_\ell\Phi)\hat{\ell}$$

Gravitational Lensing

γ as moving with velocity:

$$\mathbf{v} \approx c\,\hat{\ell}$$

The force is

$$\mathbf{F} \propto -\nabla\Phi$$

Decompose:

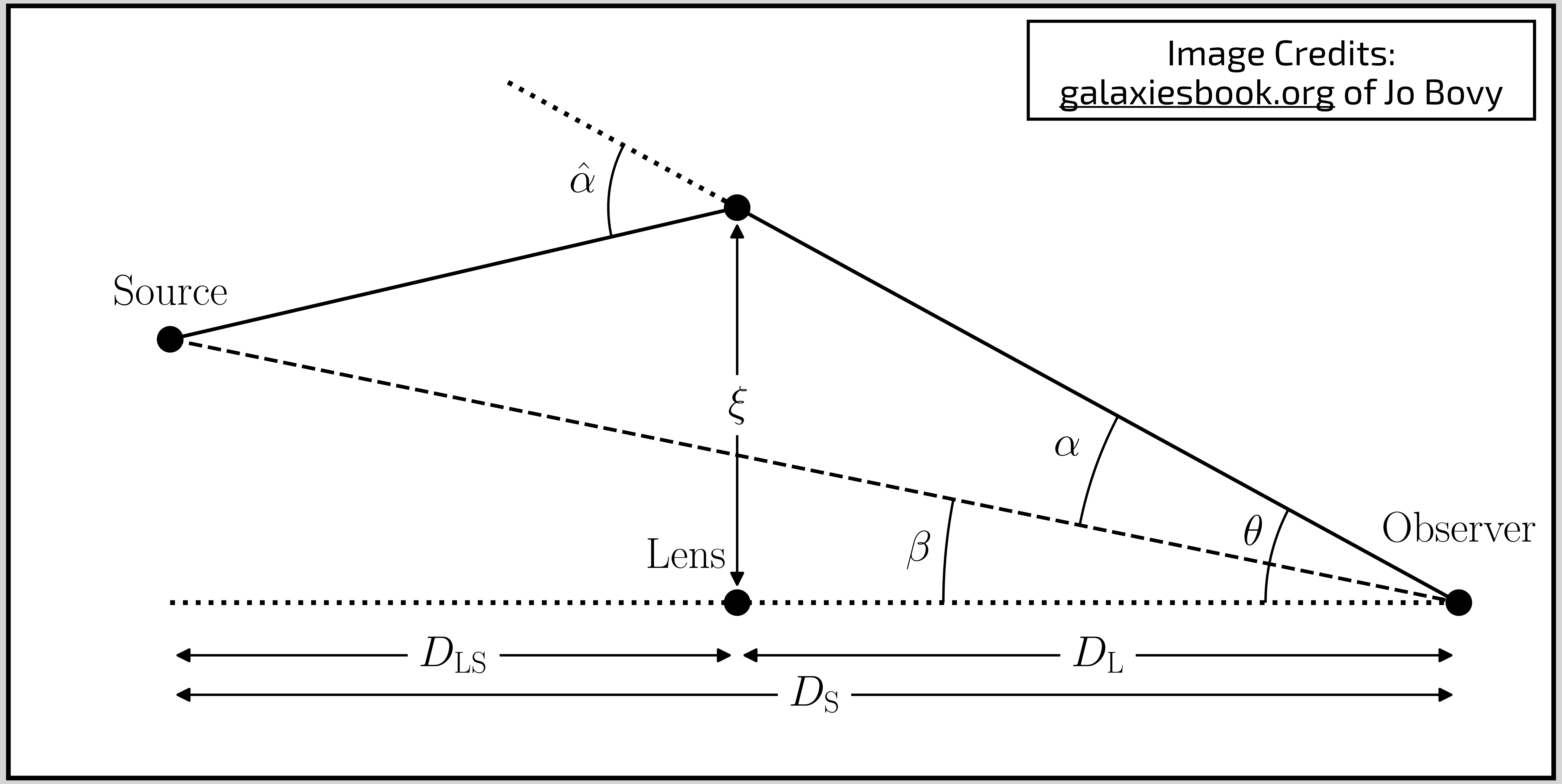
changes direction

changes speed

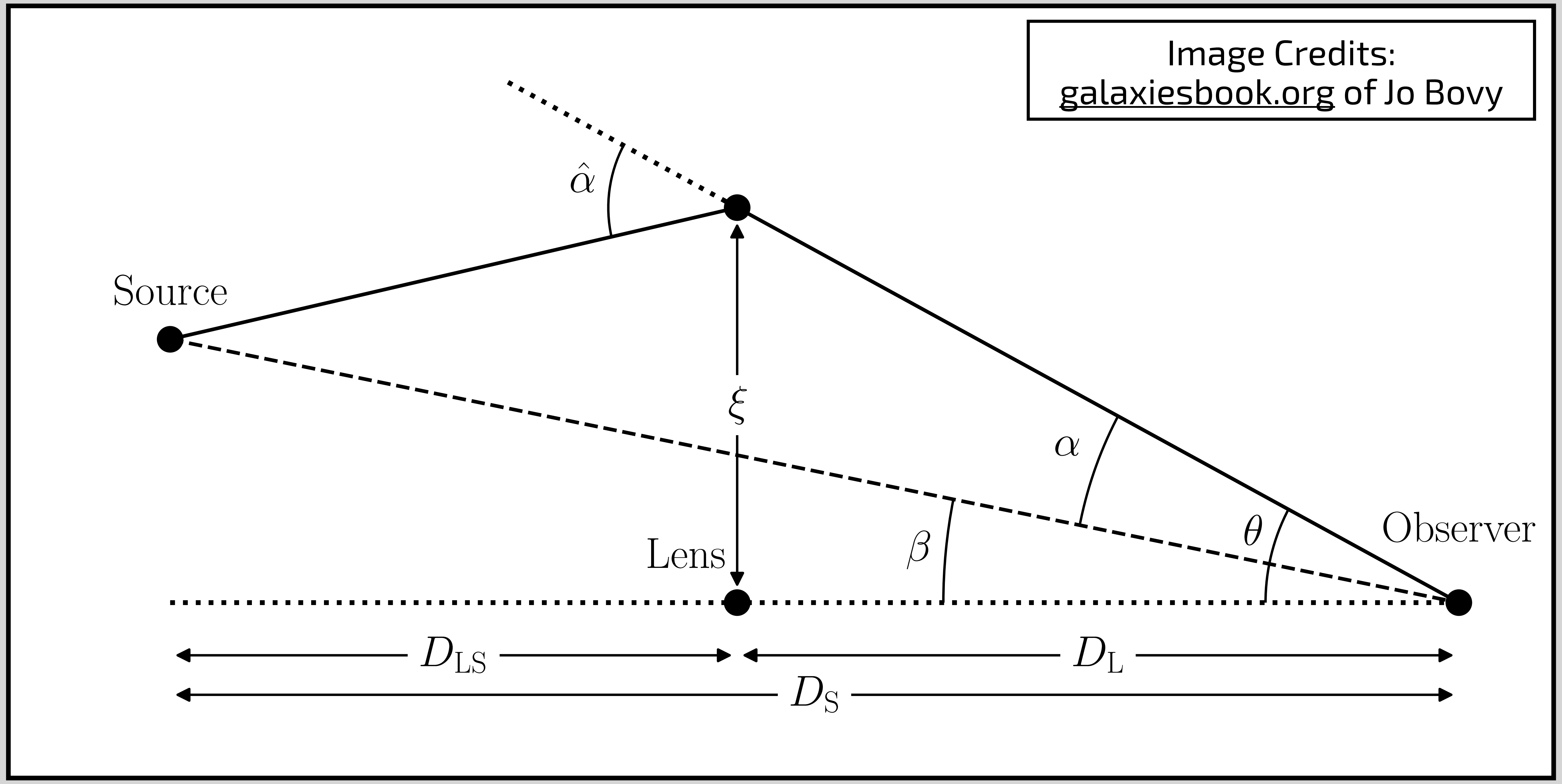
$$\nabla\Phi = (\nabla_\perp\Phi) + (\partial_\ell\Phi)\hat{\ell}$$

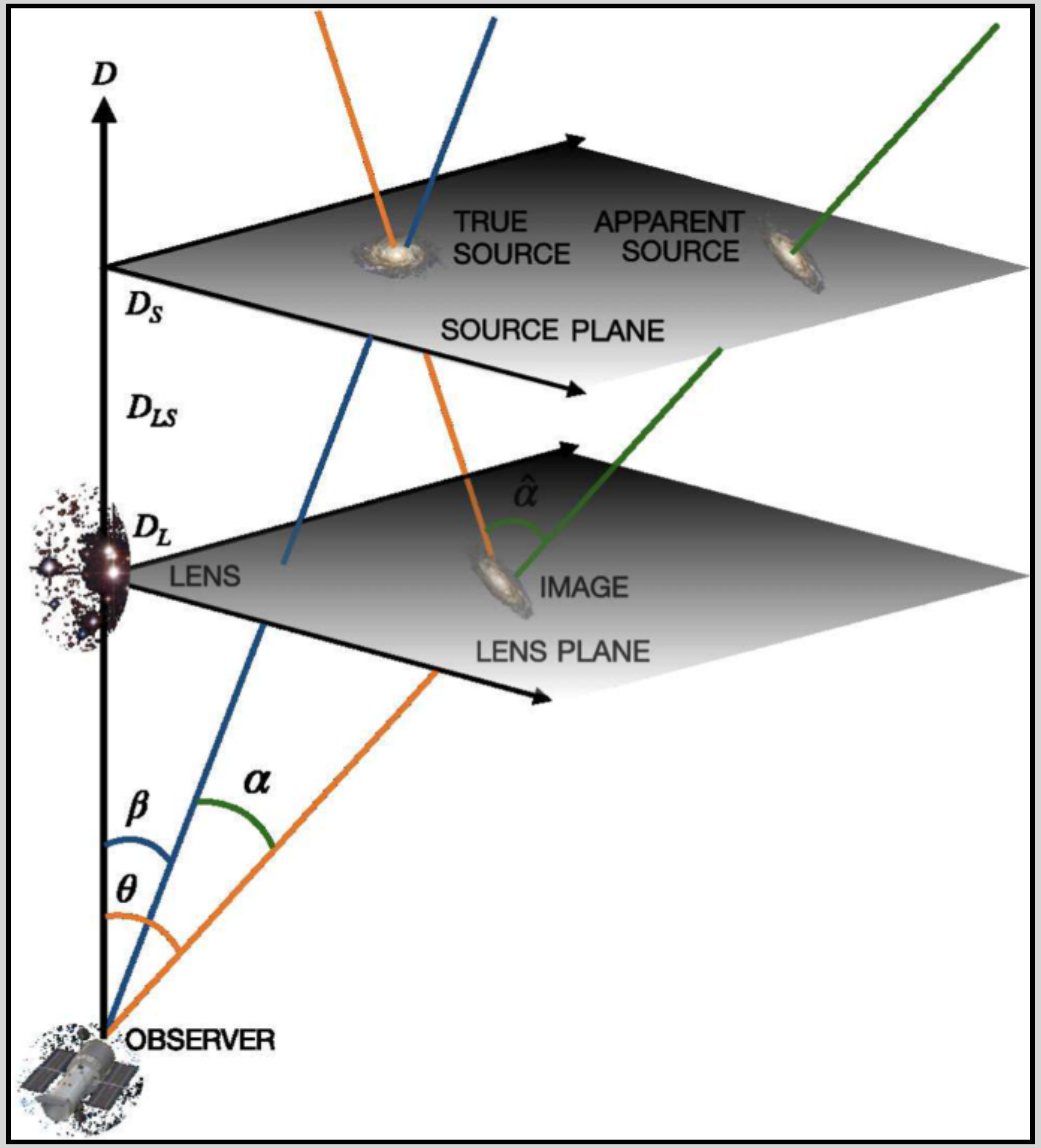
bending ($\hat{\alpha}$) $\approx$ lensing

Gravitational Lensing

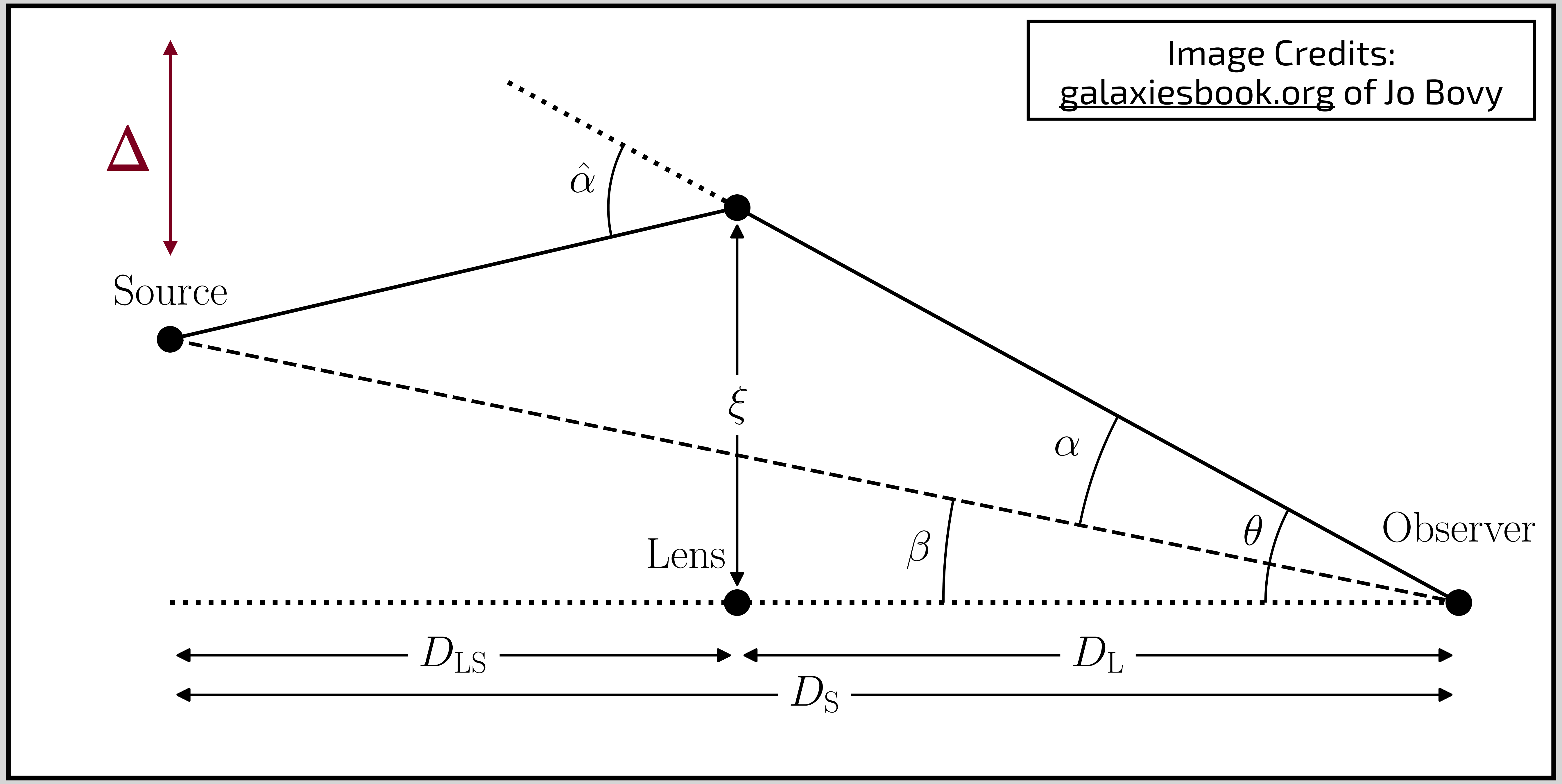


Gravitational Lensing





Gravitational Lensing



Lens equation:

$$\beta = \theta - \alpha(\theta)$$

$$\beta = \theta - \nabla_{\theta}\psi(\theta)$$

Gravitational Lensing

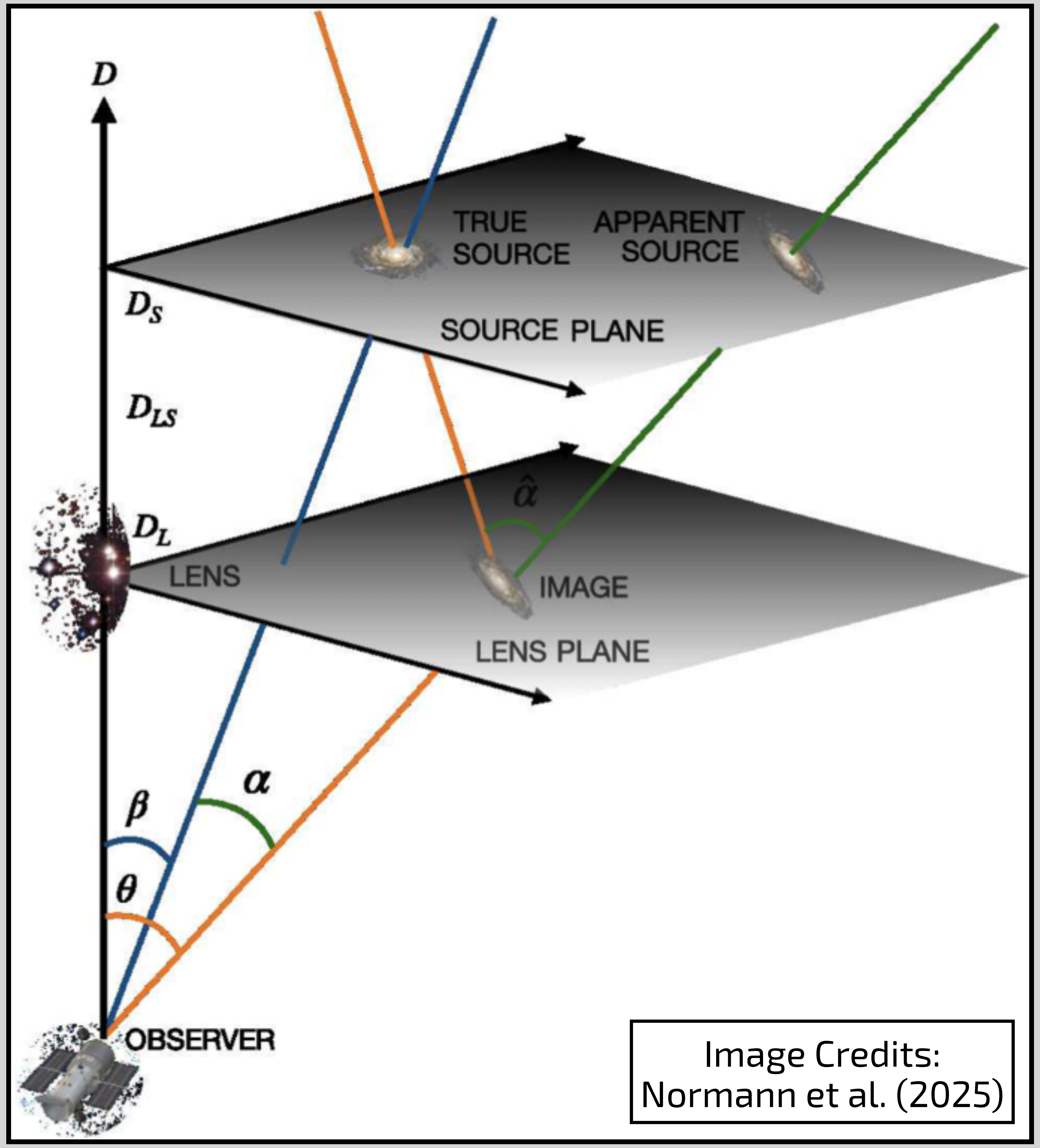


Image Credits:
Normann et al. (2025)

Lens equation:

$$\beta = \theta - \alpha(\theta)$$

$$\beta = \theta - \nabla_\theta \psi(\theta)$$

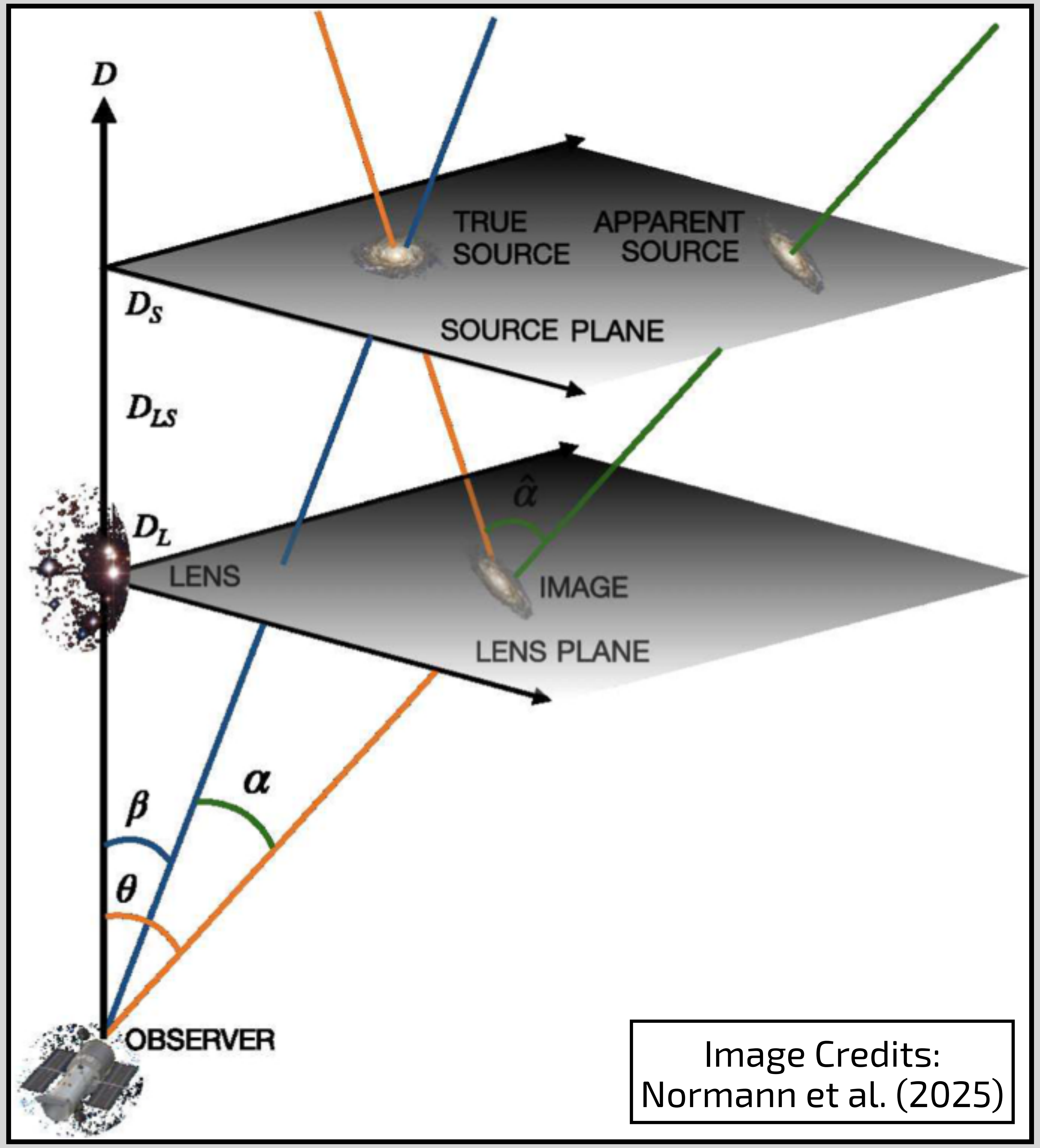
Jacobian

(magnification matrix):

$$A \equiv \partial \beta / \partial \theta$$

$$d\theta = A^{-1} d\beta$$

Gravitational Lensing



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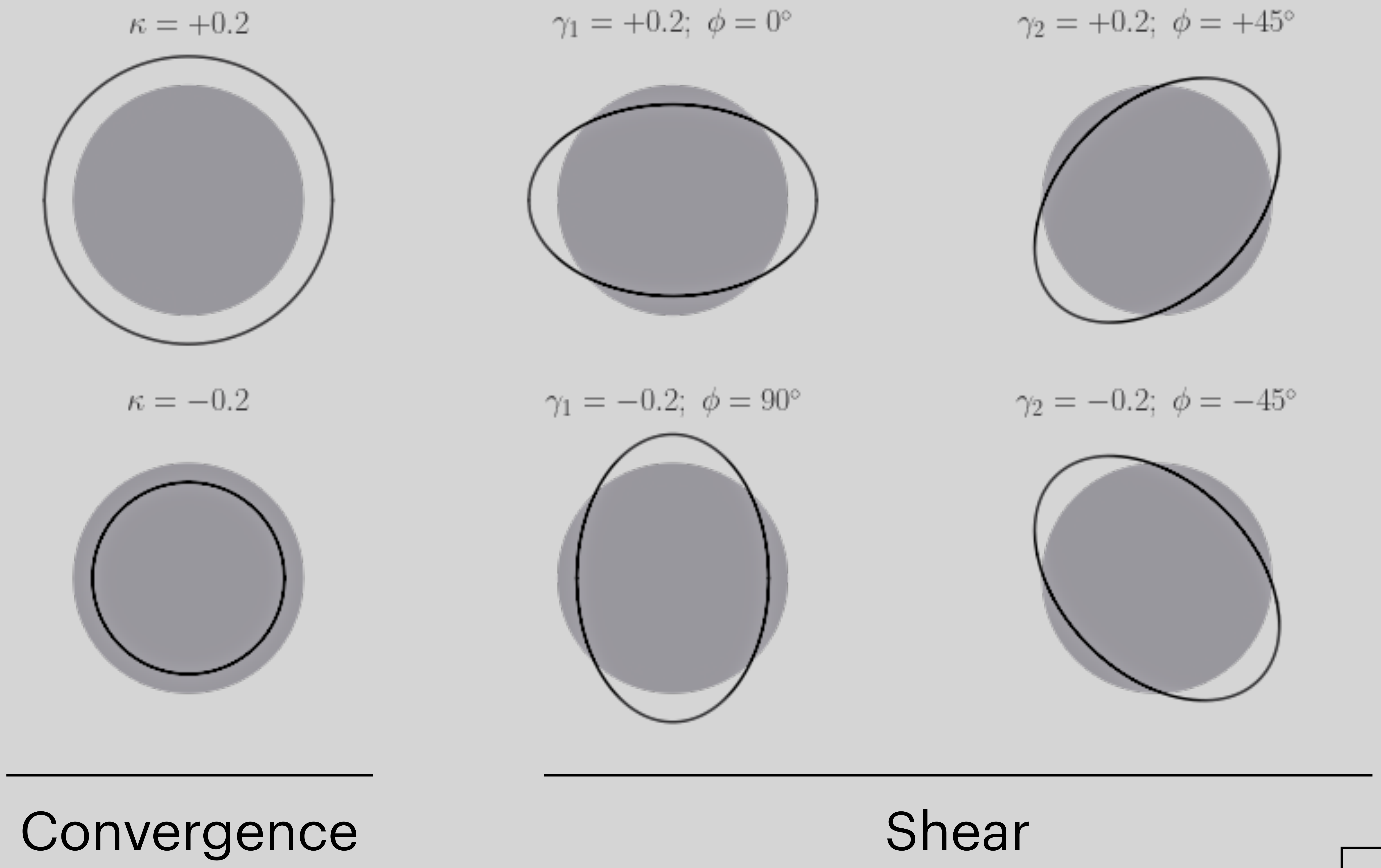
Normann et al. (2025)

$$\mathcal{A}_{ij} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\mathcal{A}_{ij} = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

$$\mathcal{A}_{ij} = 1-\kappa \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$

Gravitational Lensing



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Accurate cosmological emulator for the probability distribution function of gravitational lensing of point sources

<u>ACE_LENSING</u>

Emulator

Parameters

$$\Omega_m \rightarrow (0.2 - 0.4)$$

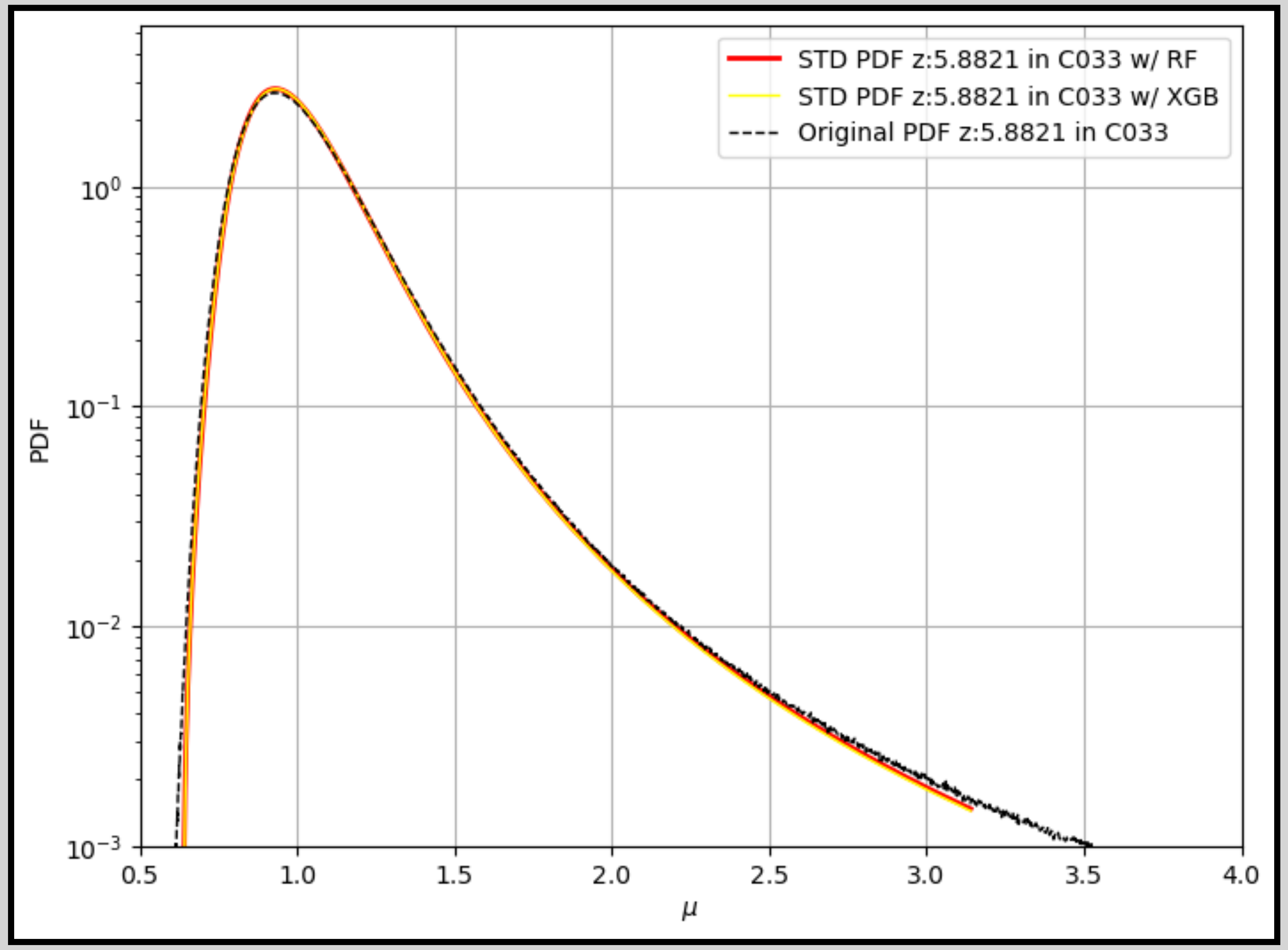
$$h \rightarrow (0.6 - 0.8)$$

$$w \rightarrow (-1.5 - -0.7)$$

$$\sigma_8 \rightarrow (0.7 - 0.9)$$

$$z \rightarrow (0.3 - 6)$$

Statistics



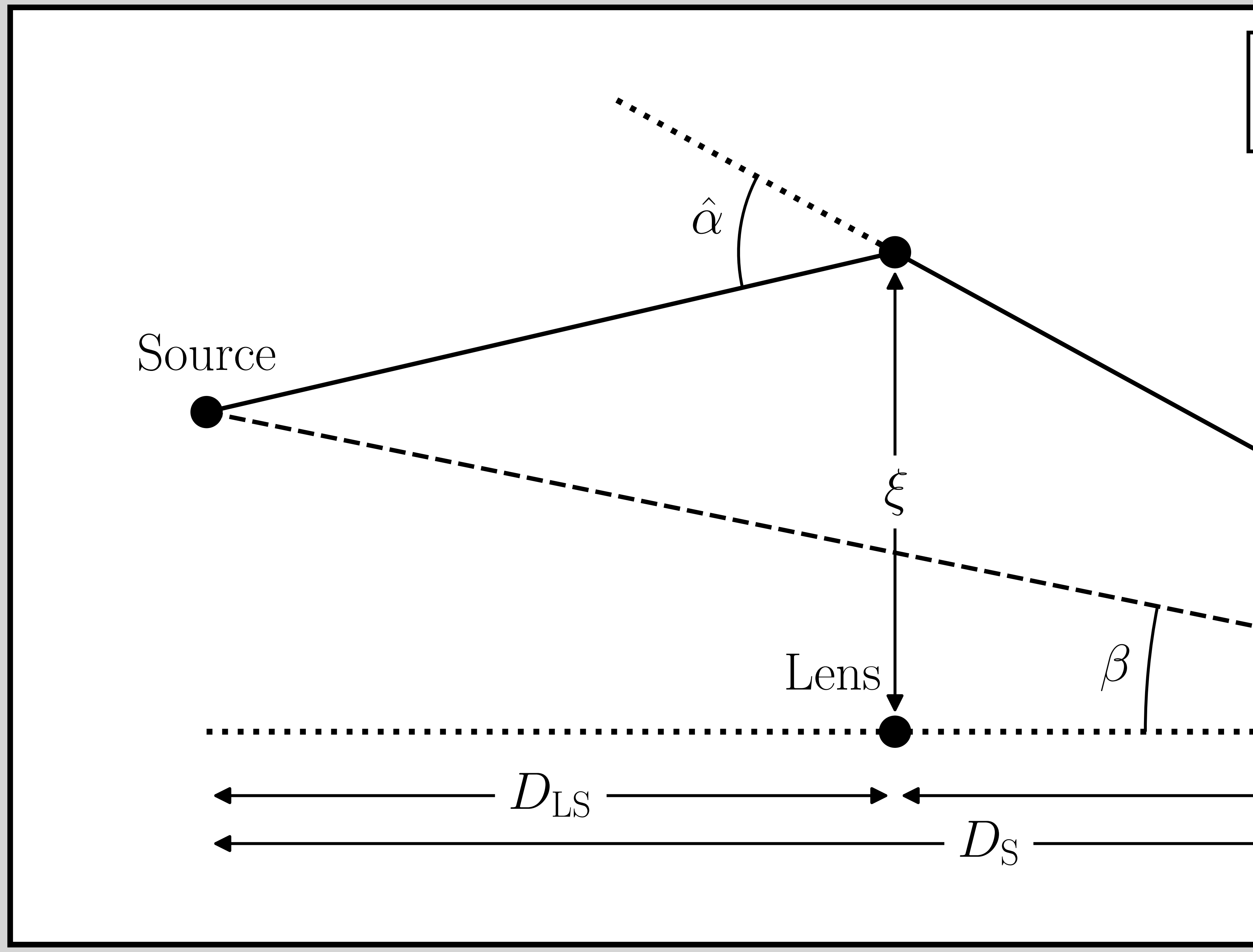
Emulator

	Traditional Methods	Emulator-Based Method
Use Case	Detail modeling	Fast parameter estimation
Computational Cost	High	Low
Speed	Slow	Fast
Flexibility	Depends on simulation or model	High
Accuracy	High	Good approximation

Accurate cosmological emulator for the probability distribution function of gravitational lensing of point sources

ACE_LENSING

Weak Gravitaional Lensing



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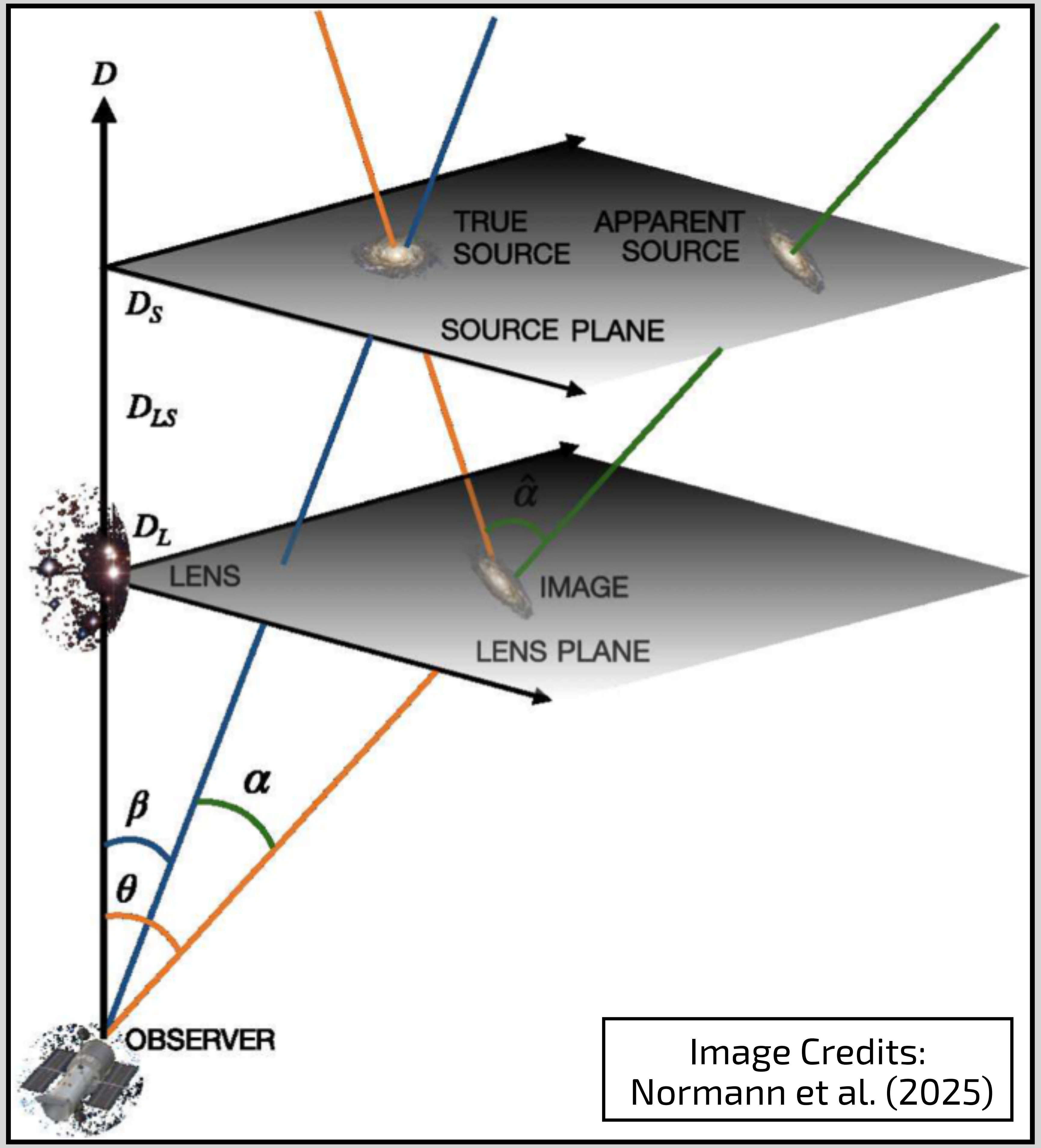

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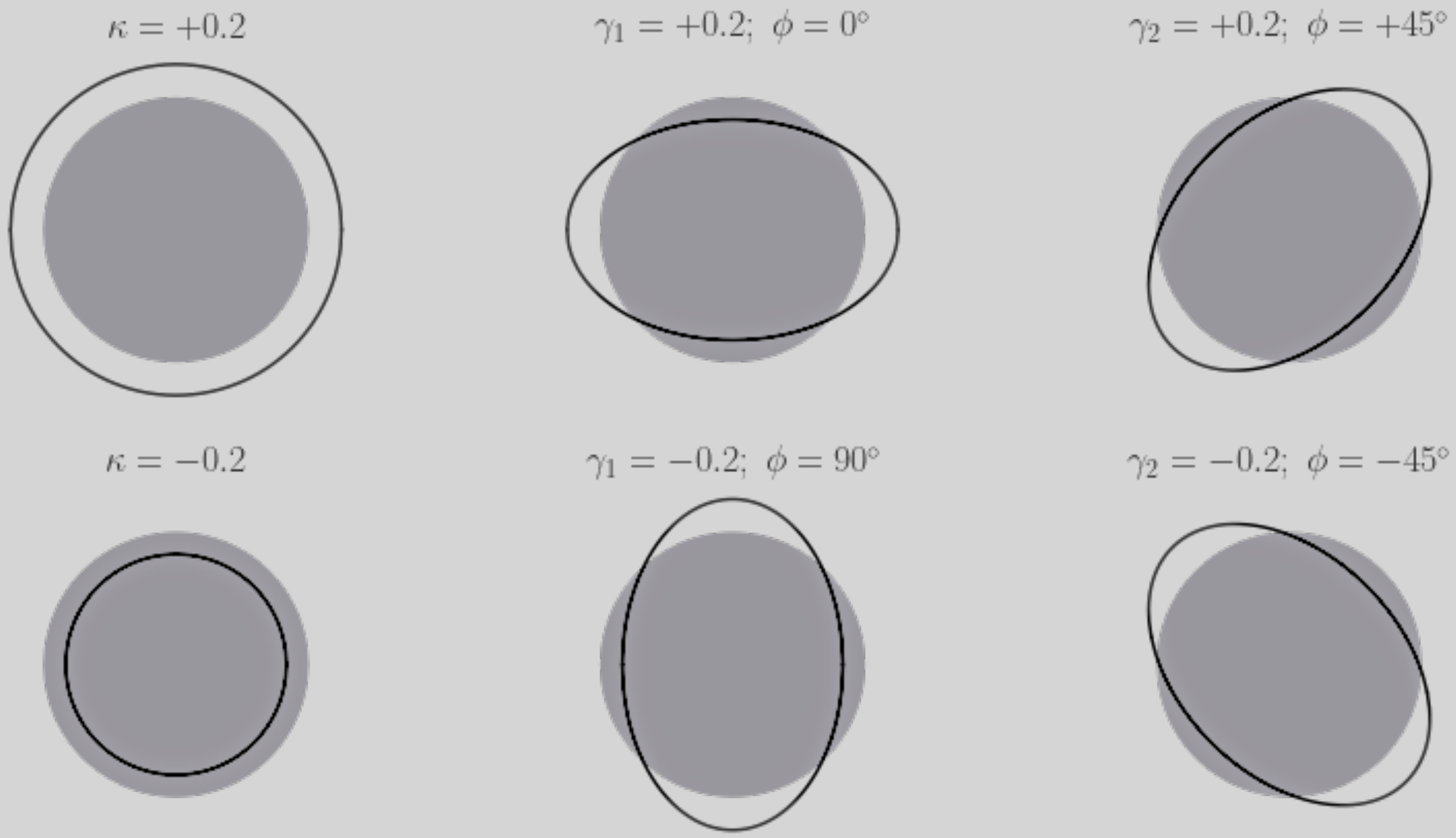
Weak Gravitaional Lensing



Convergence

Shear

$$\mu = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

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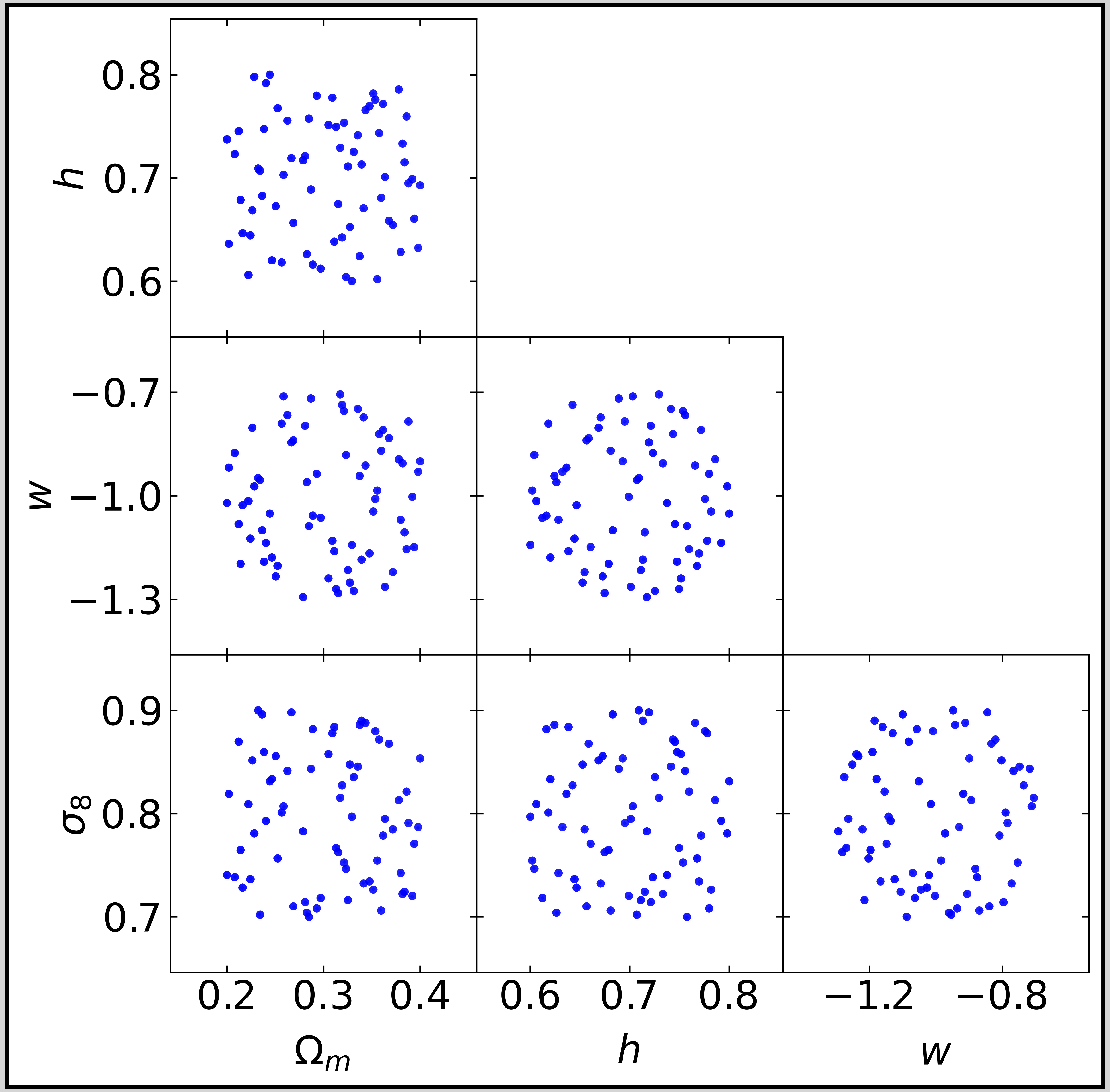
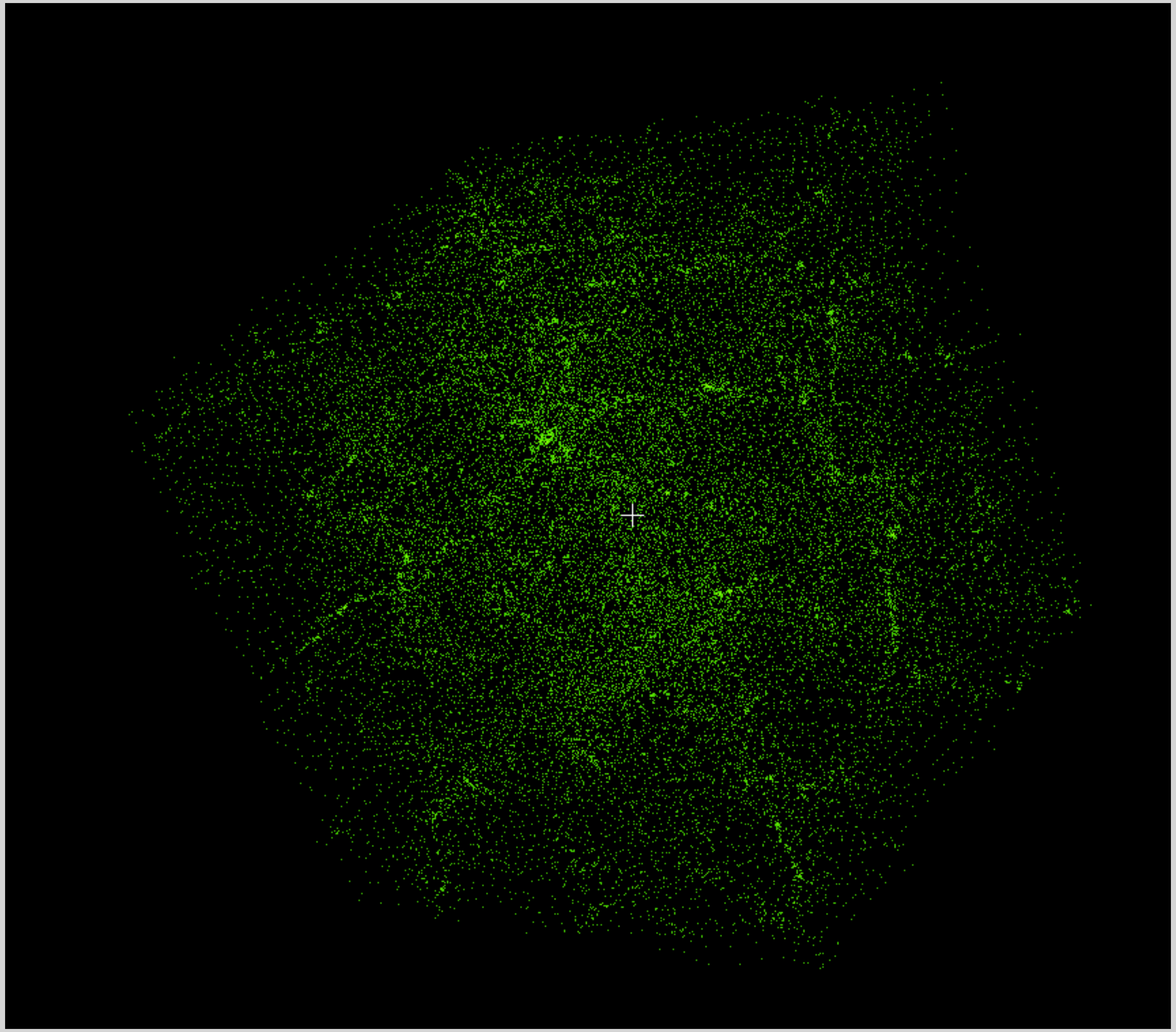
Parameter Space

Ω_m	$[0.2, 0.4]$
σ_8	$[0.7, 0.9]$
w	$[-1.5, -0.7]$
h	$[0.6, 0.8]$

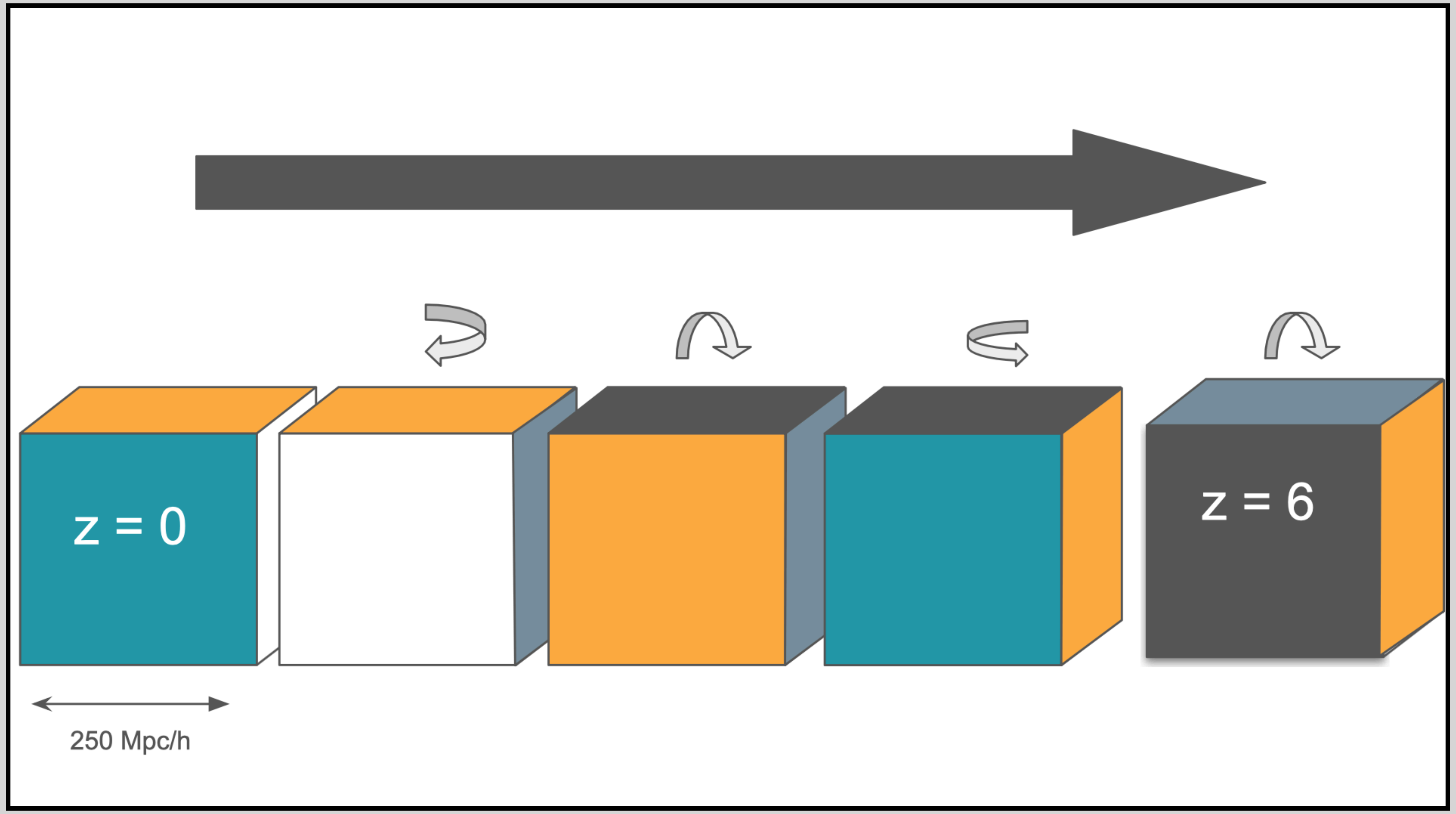


Simulations

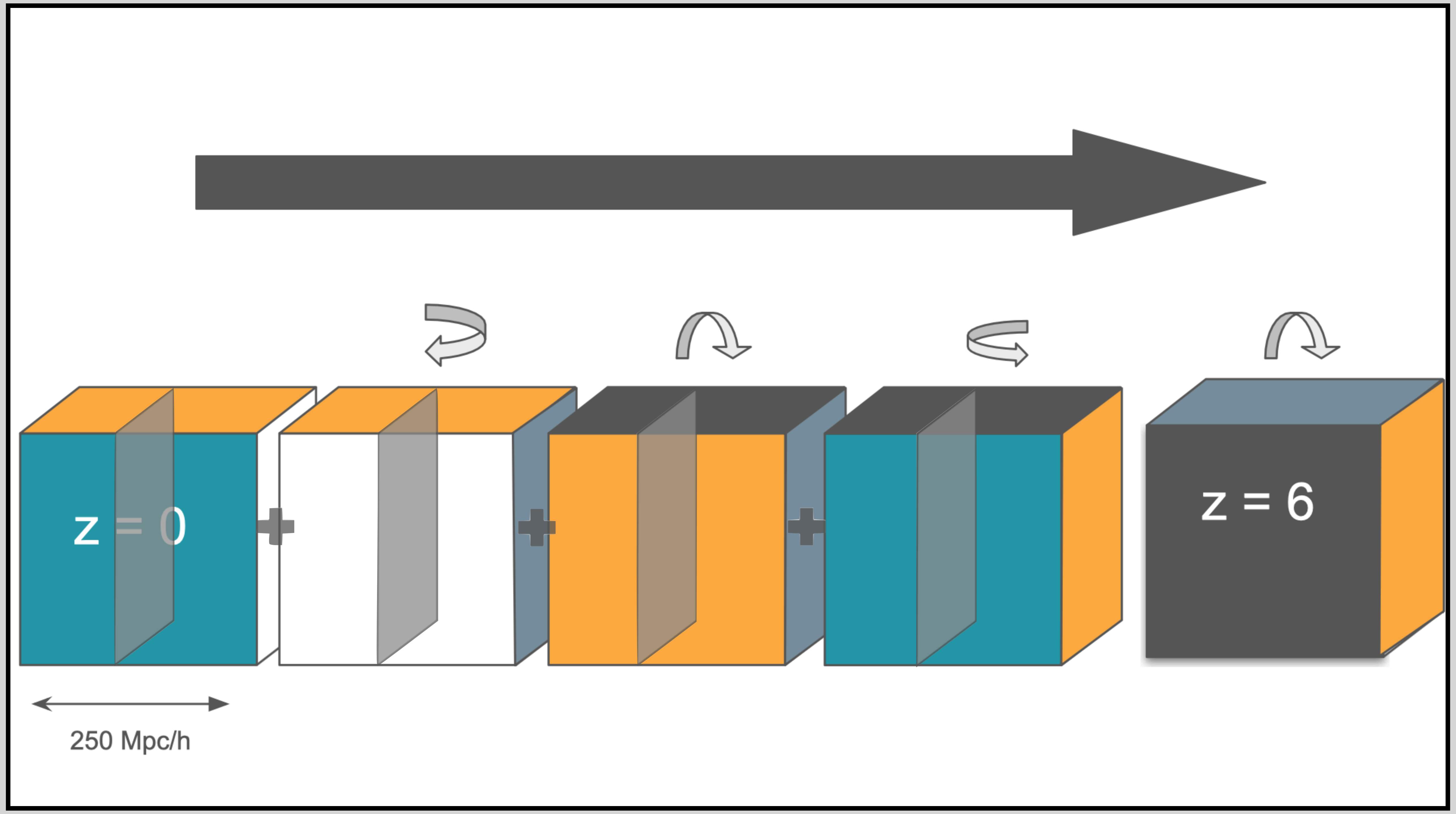


Simulation parameters	
Box size L	$250\,\mathrm{Mpc}\,h^{-1}$
Number of particles N	4×640^3
Particle-Mesh grid	2048^3
Initial redshift	49
Softening length	$6\,\mathrm{kpc}\,h^{-1}$
Particle mass	$4.1\,\Omega_\mathrm{m} \times 10^9\,M_\odot\,h^{-1}$

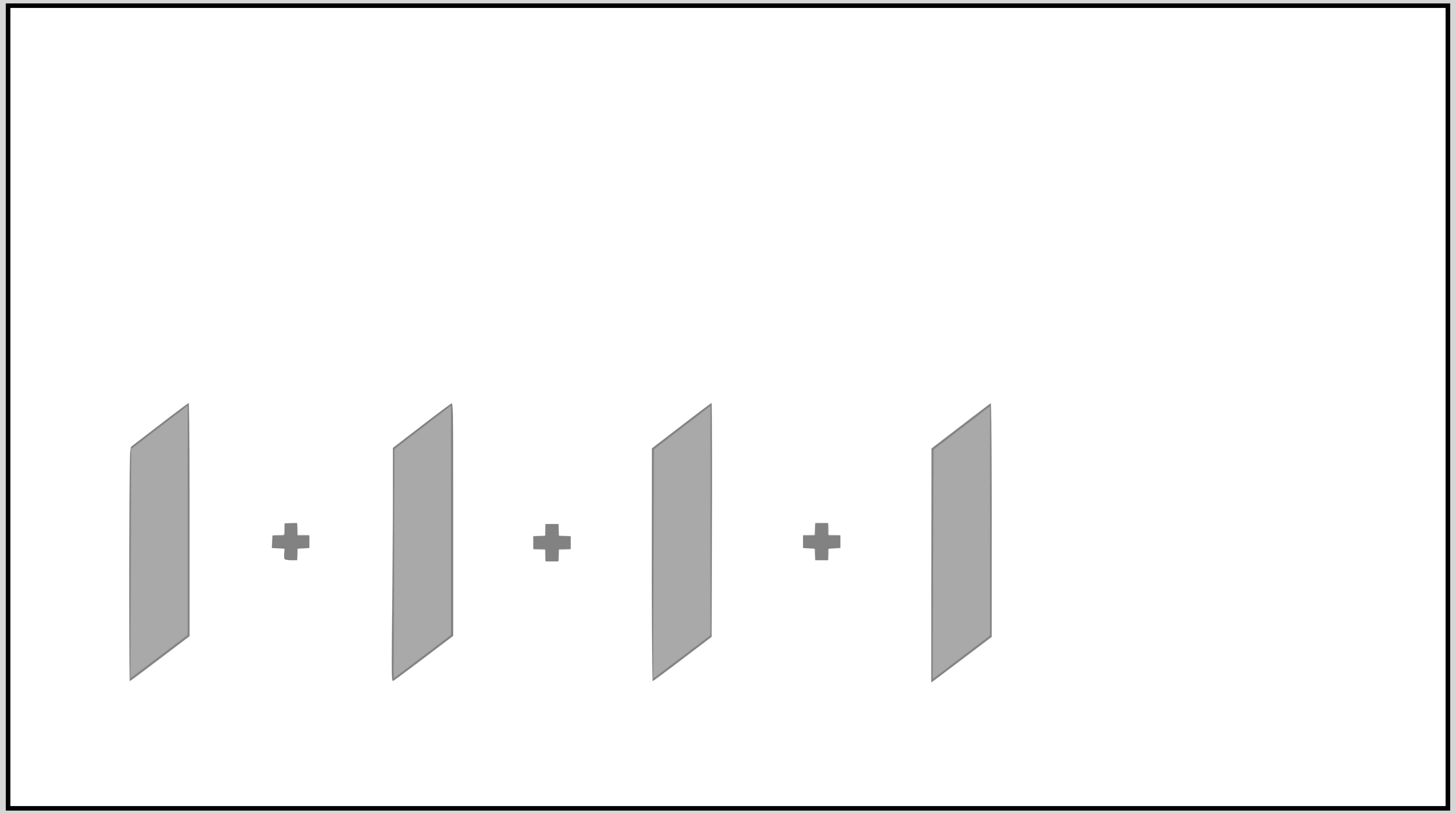
Past Light Cones



Past Light Cones



Past Light Cones



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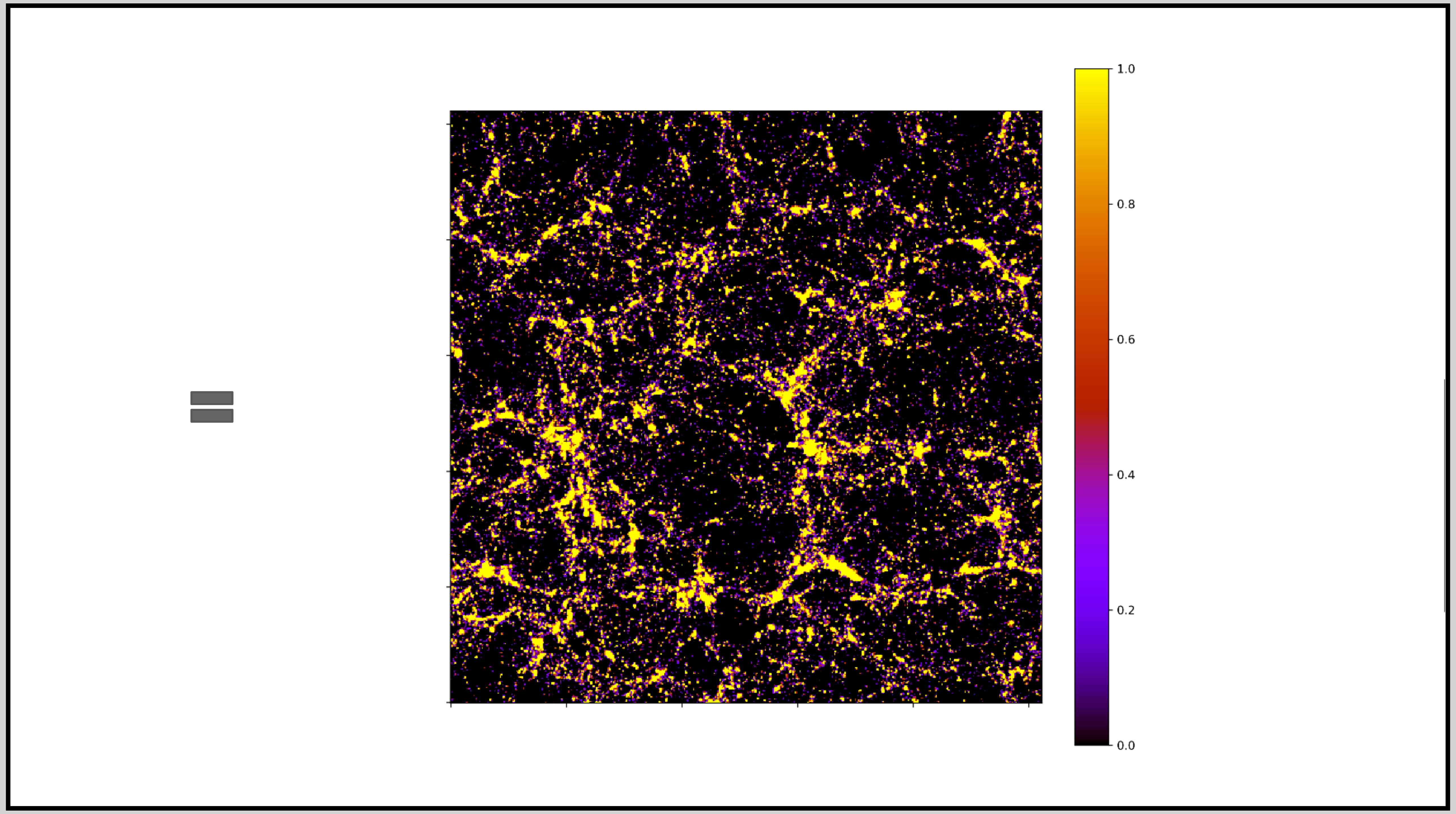


Magnification Maps

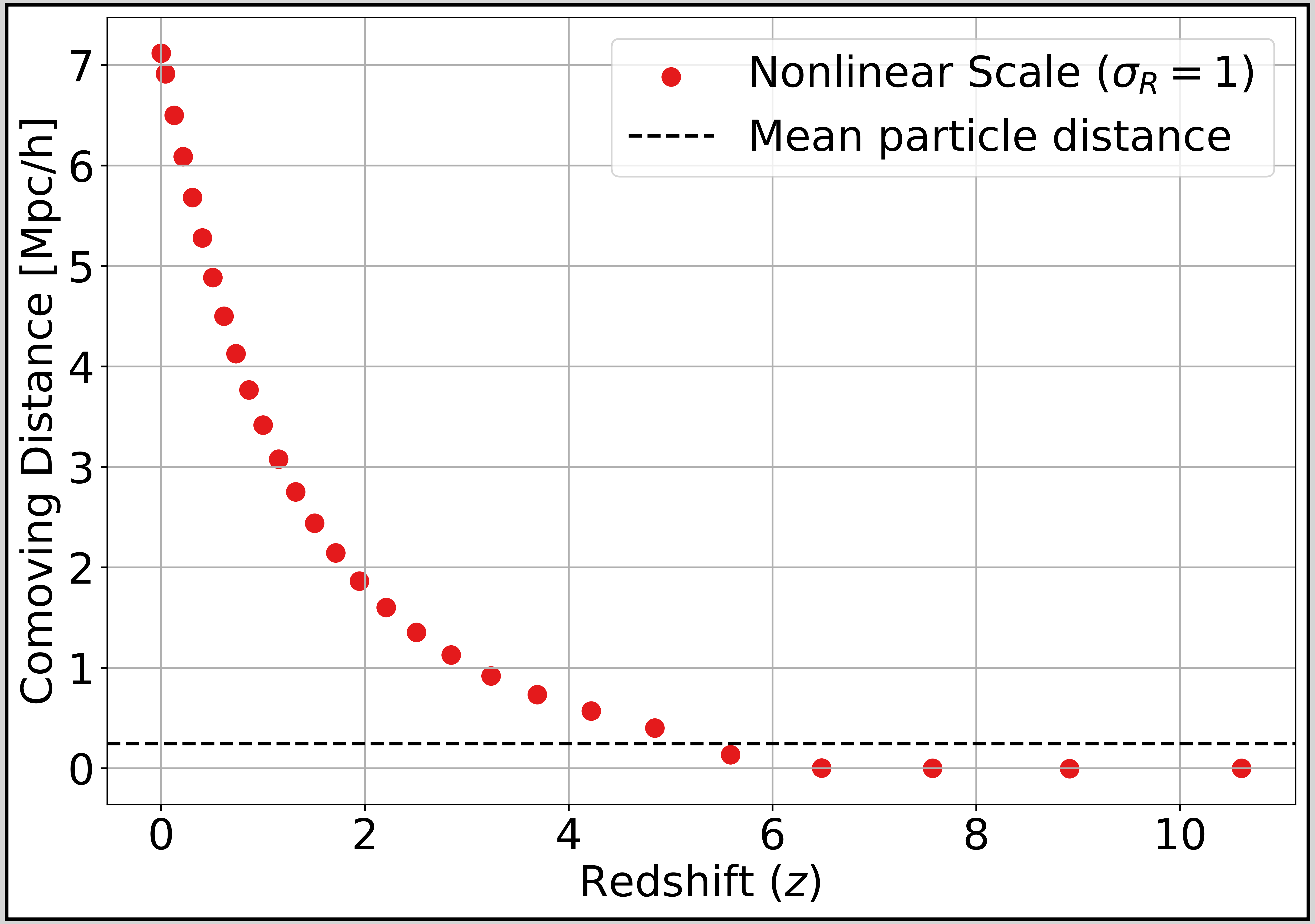
$$\kappa(\mathbf{x}) = \frac{4\pi G}{c^2} \sum_i W(z_i, z_s) \left[\Sigma_i(\mathbf{x}) - \langle \Sigma_i \rangle\right] (1 + z_i)^2 \frac{G(z_i)}{G(z_{\text{snap}})}$$

$$\tilde{\phi}(\mathbf{k}) = -\frac{2}{|\mathbf{k}|^2}\tilde{\kappa}(\mathbf{k})$$

$$\gamma_1 = \frac{1}{2}(\partial_{xx}\phi - \partial_{yy}\phi), \qquad \gamma_2 = \partial_{xy}\phi$$

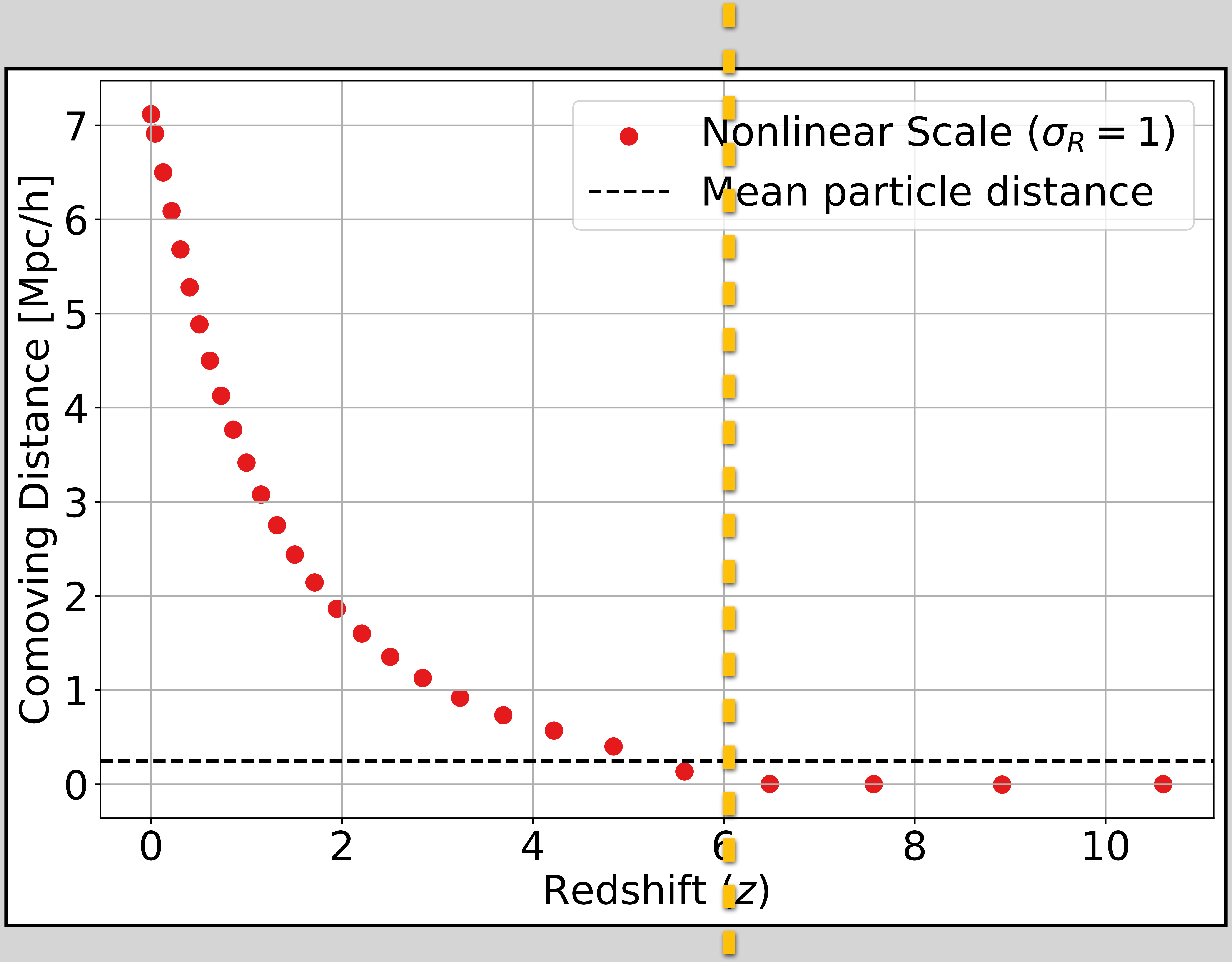
$$\mu = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

Resolution



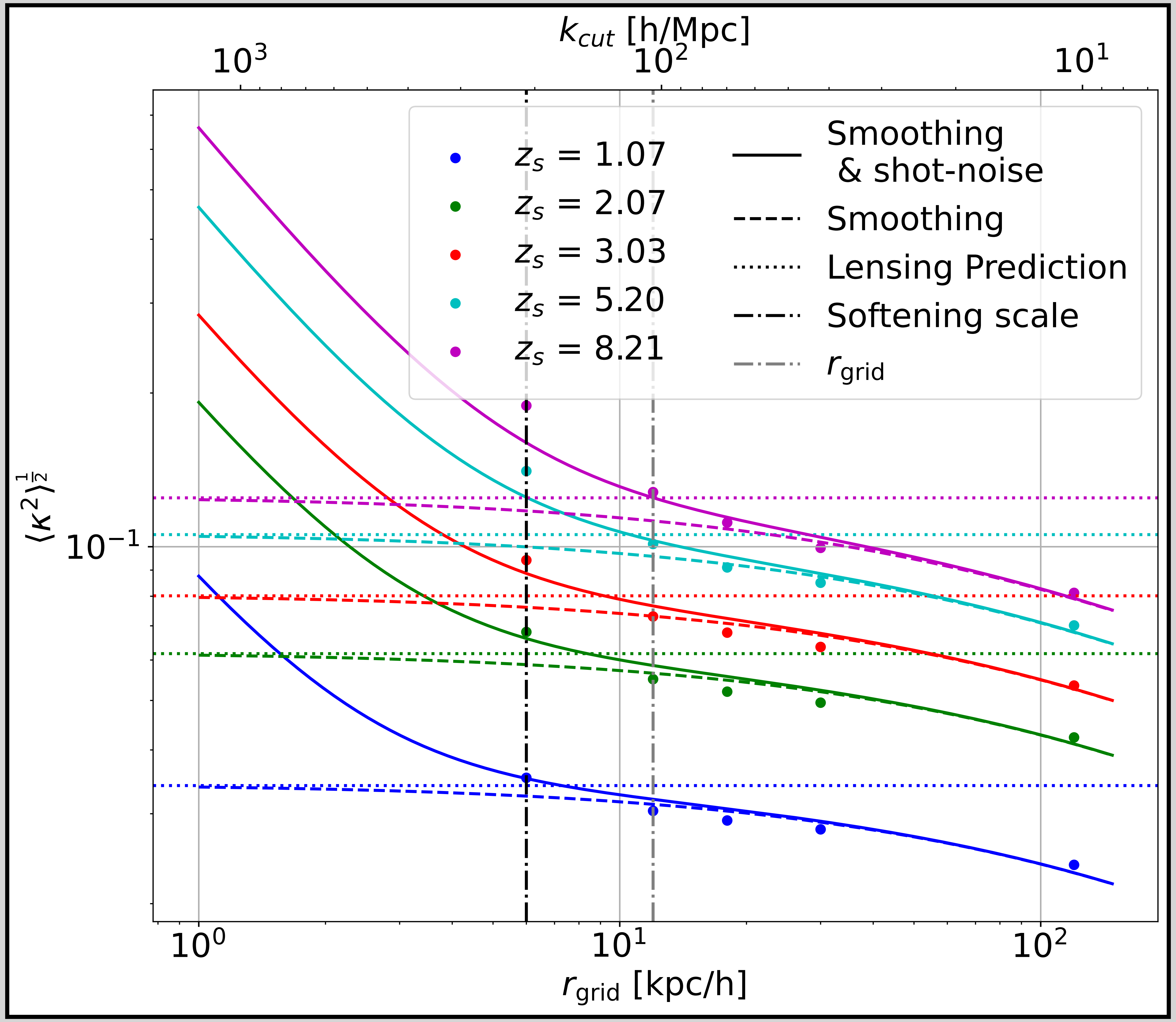
Nonlinear Scale ($\sigma_R = 1$)
Mean particle distance
Comoving Distance [Mpc/h]
Redshift (z)
0
1
2
3
4
5
6
7
0
2
4
6
8
10

Resolution

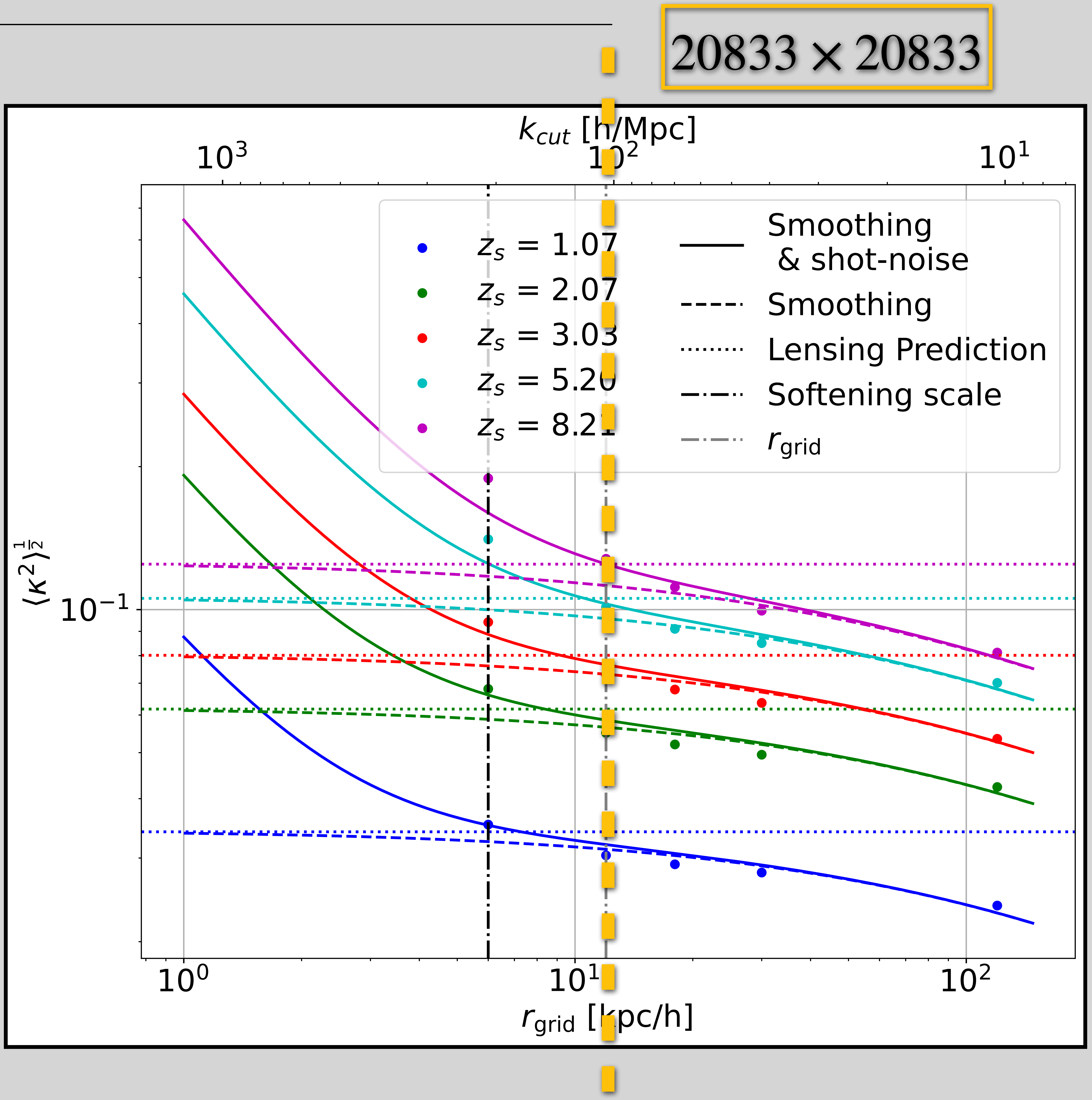


Nonlinear Scale ($\sigma_R = 1$)
Mean particle distance
Comoving Distance [Mpc/h]
Redshift (z)
0
1
2
3
4
5
6
7
0
2
4
6
8
10

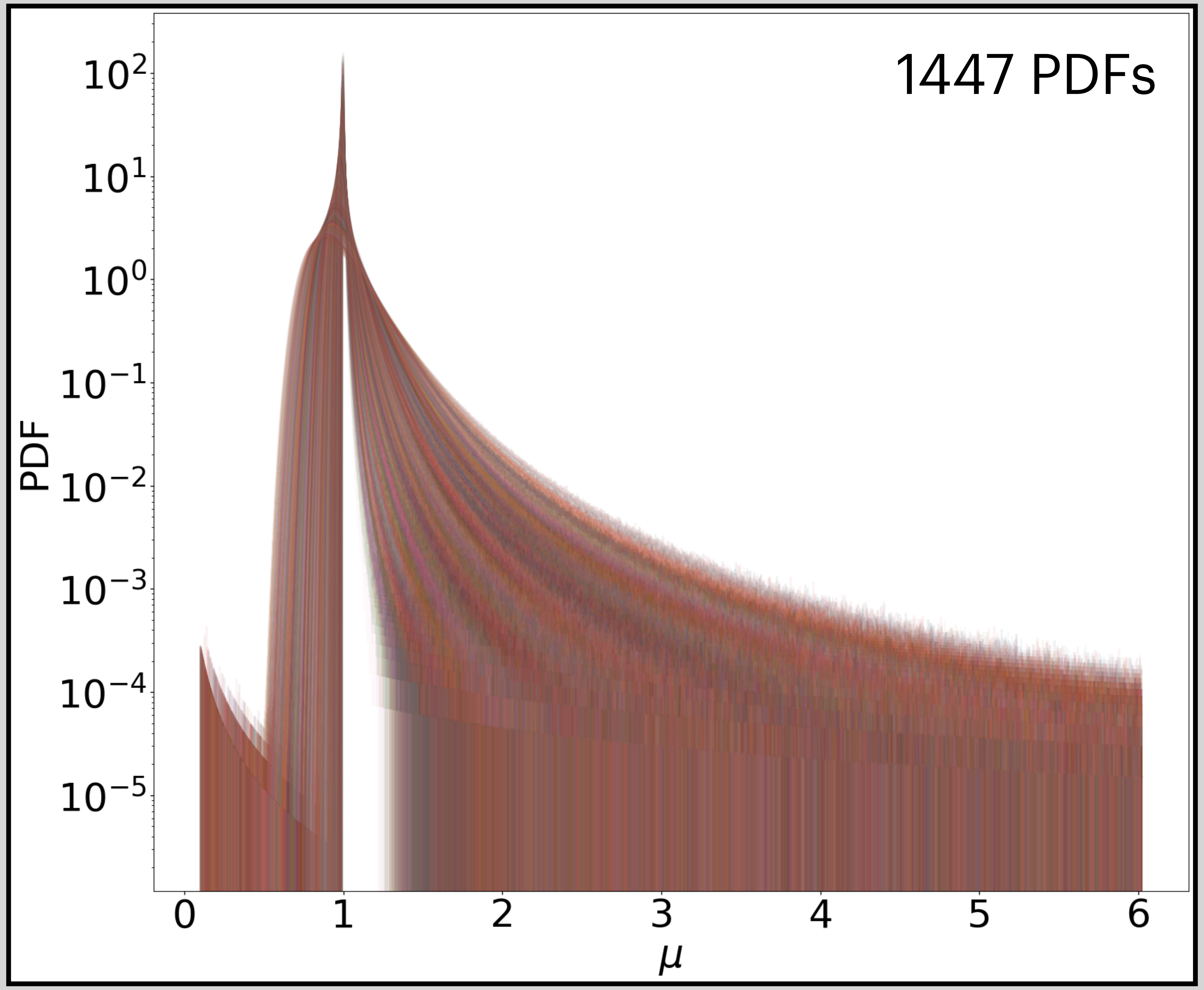
Convergence Check



Convergence Check



PDFs



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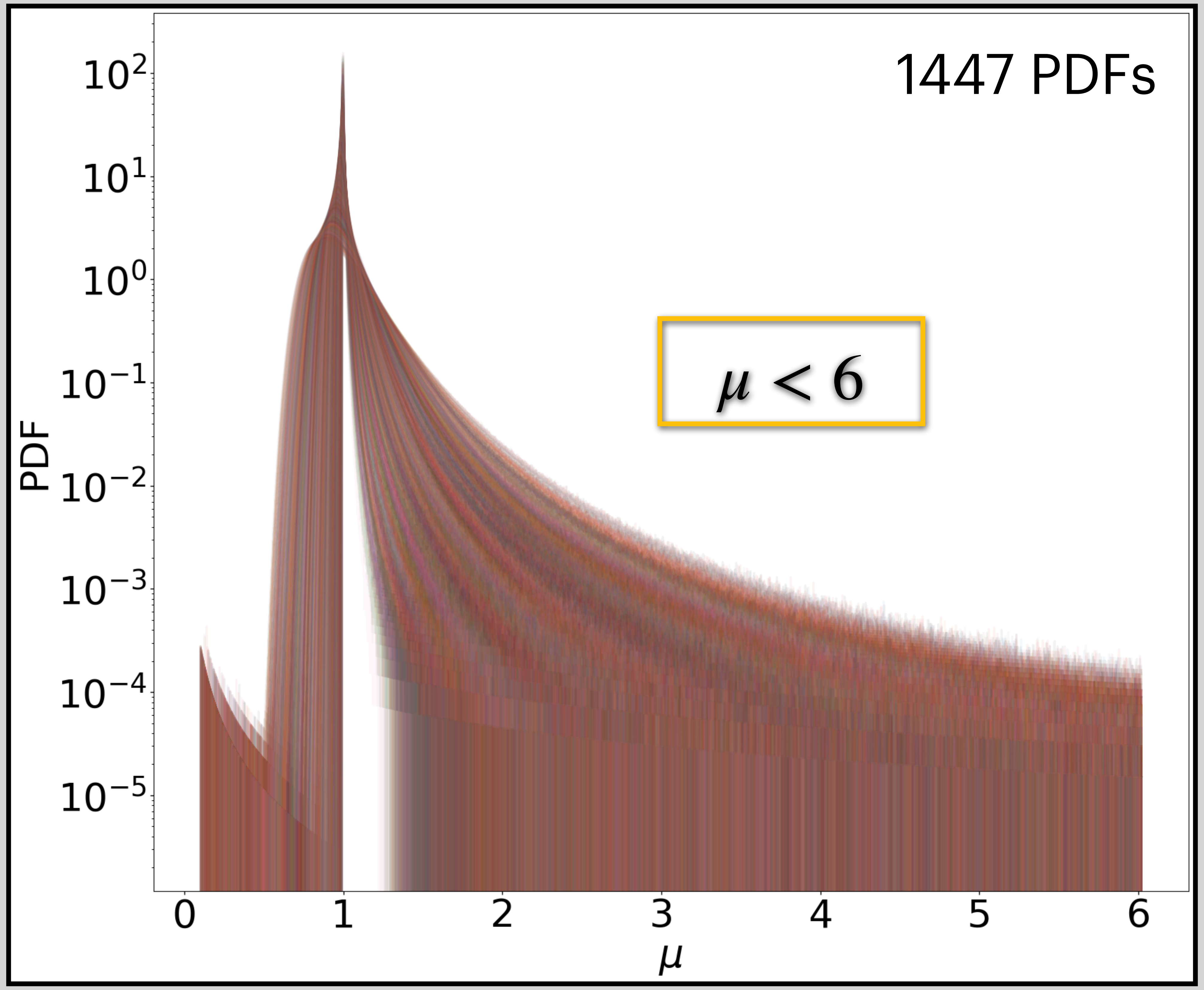

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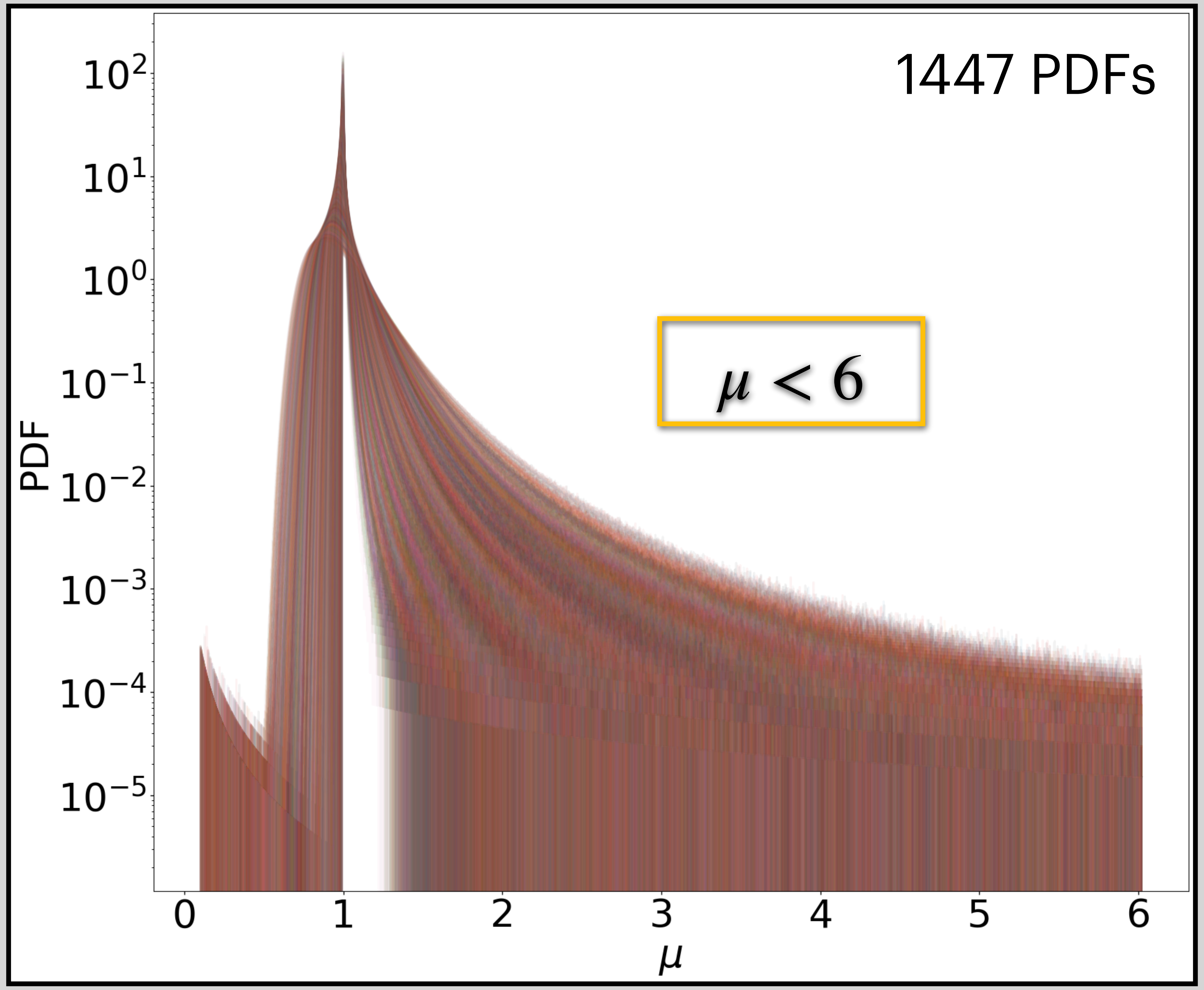
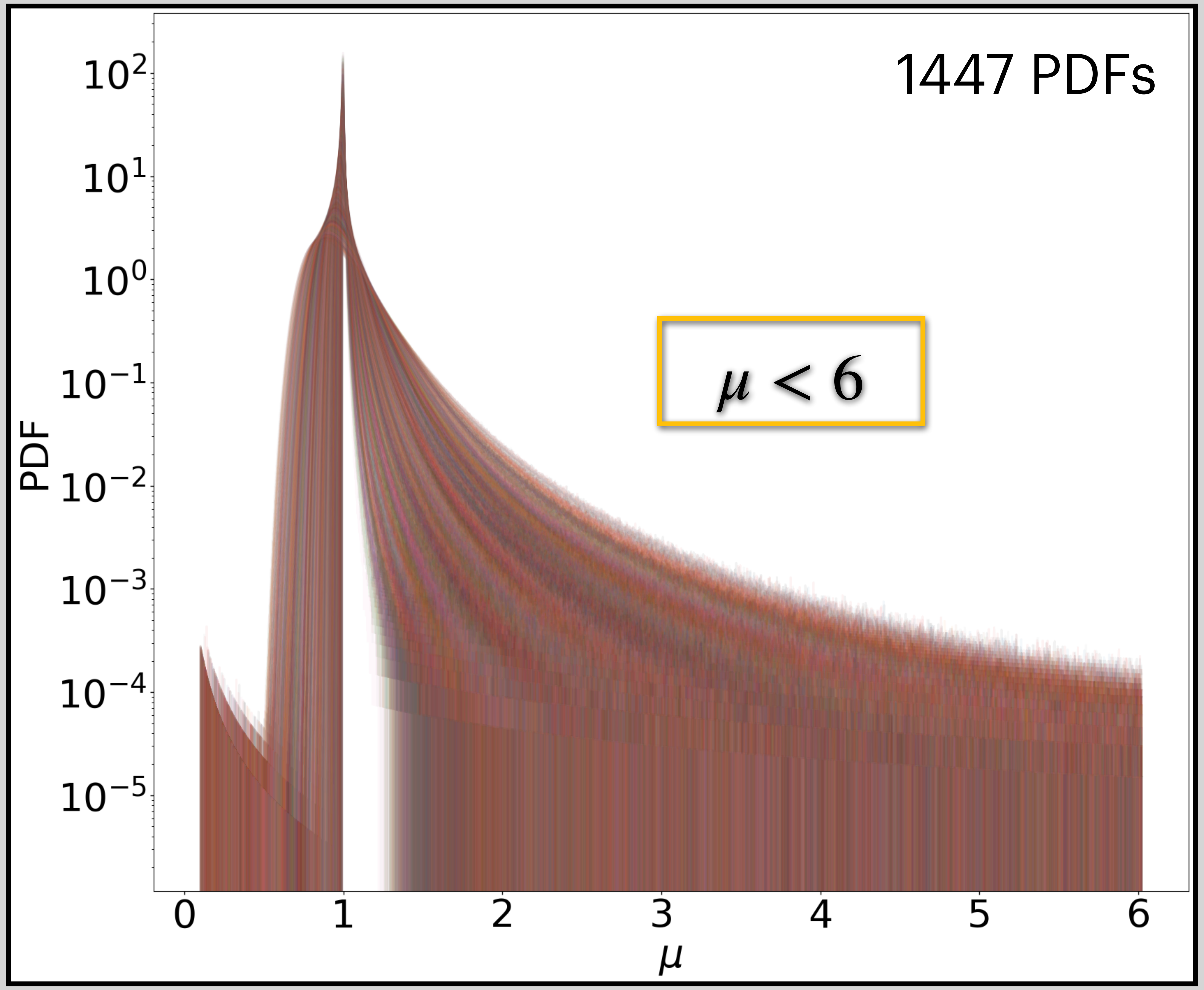
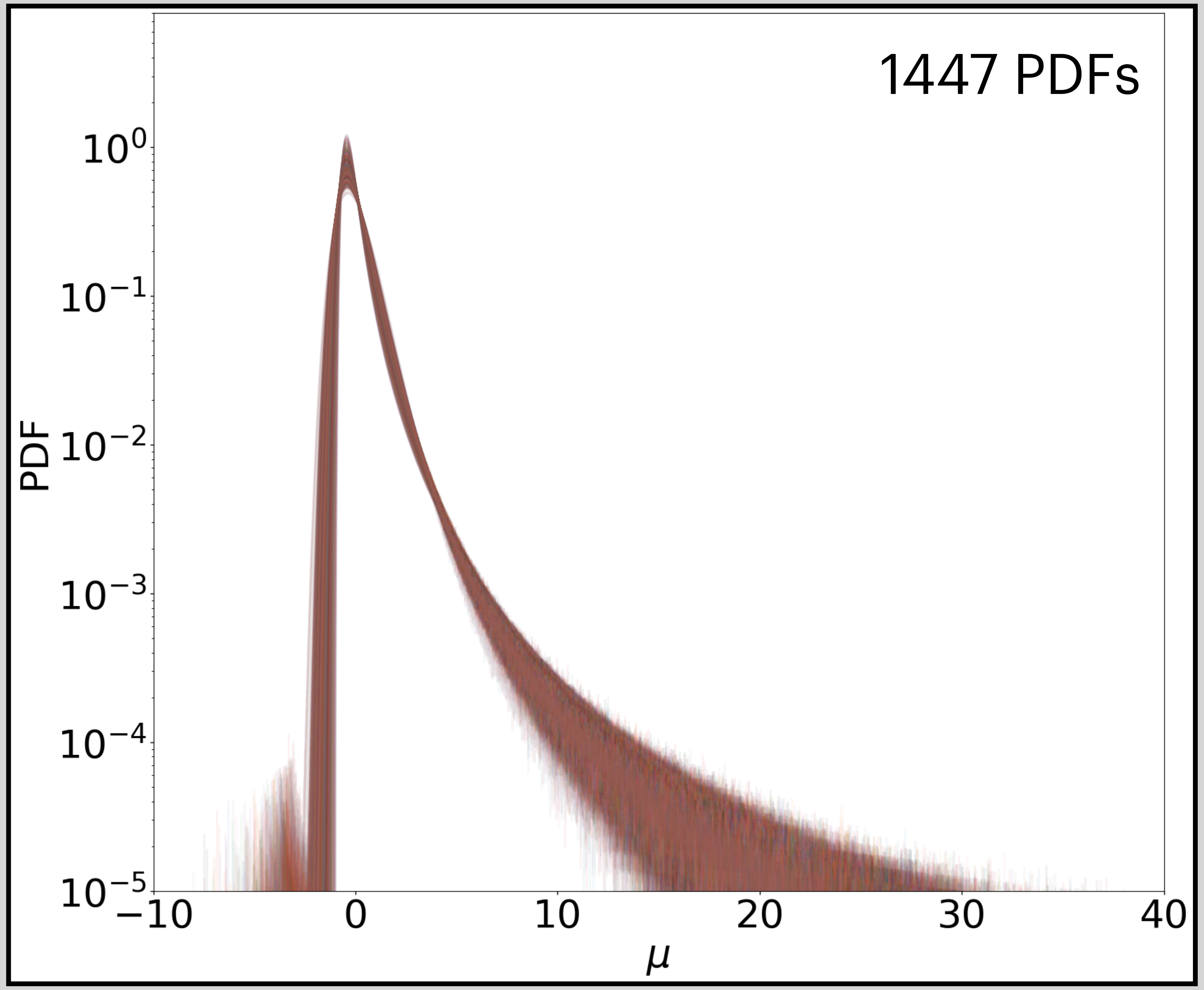


$$\frac{\mu - \bar{\mu}}{\sigma}$$

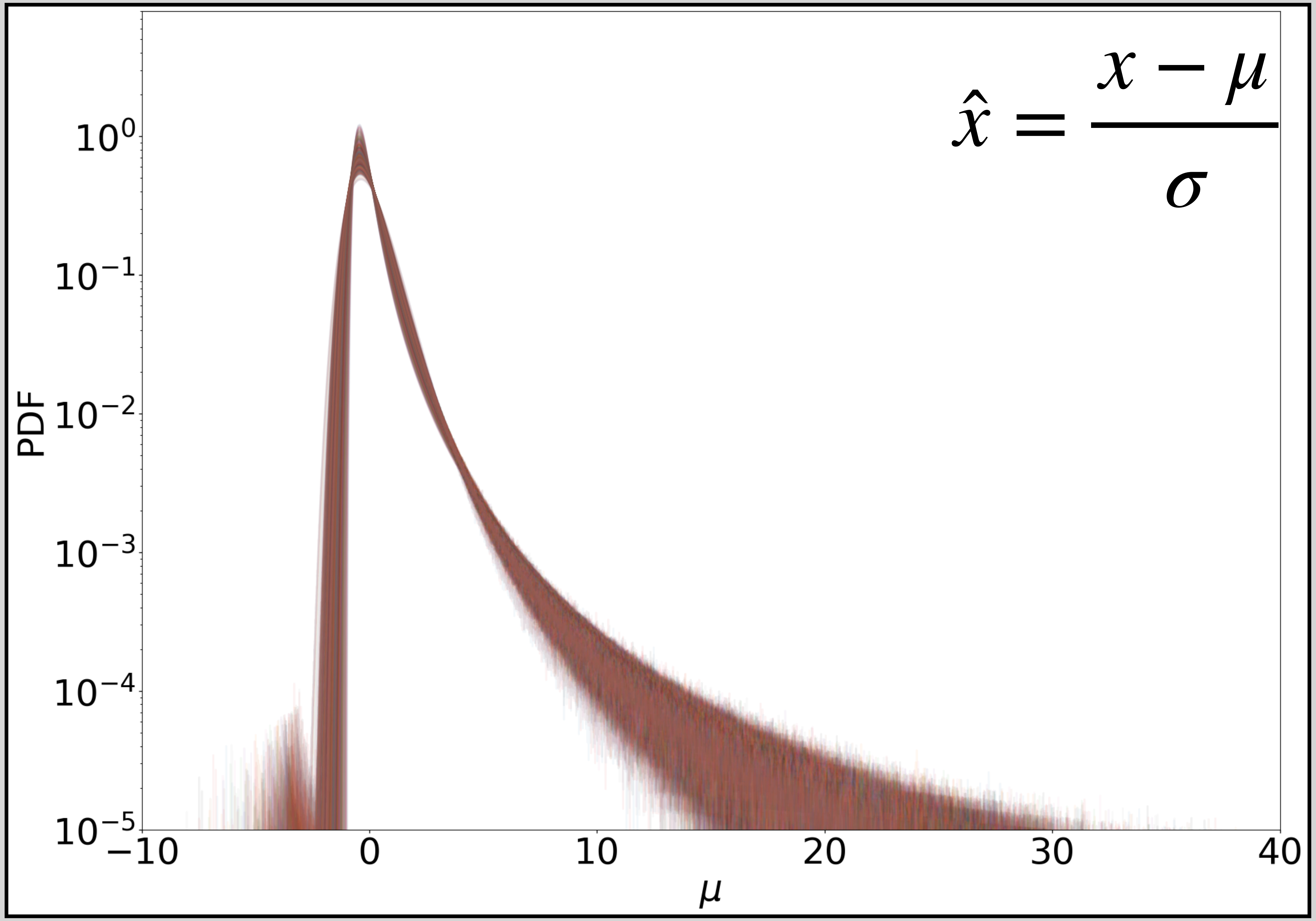
PDFs



$$\frac{\mu - \bar{\mu}}{\sigma}$$



Standardized PDFs



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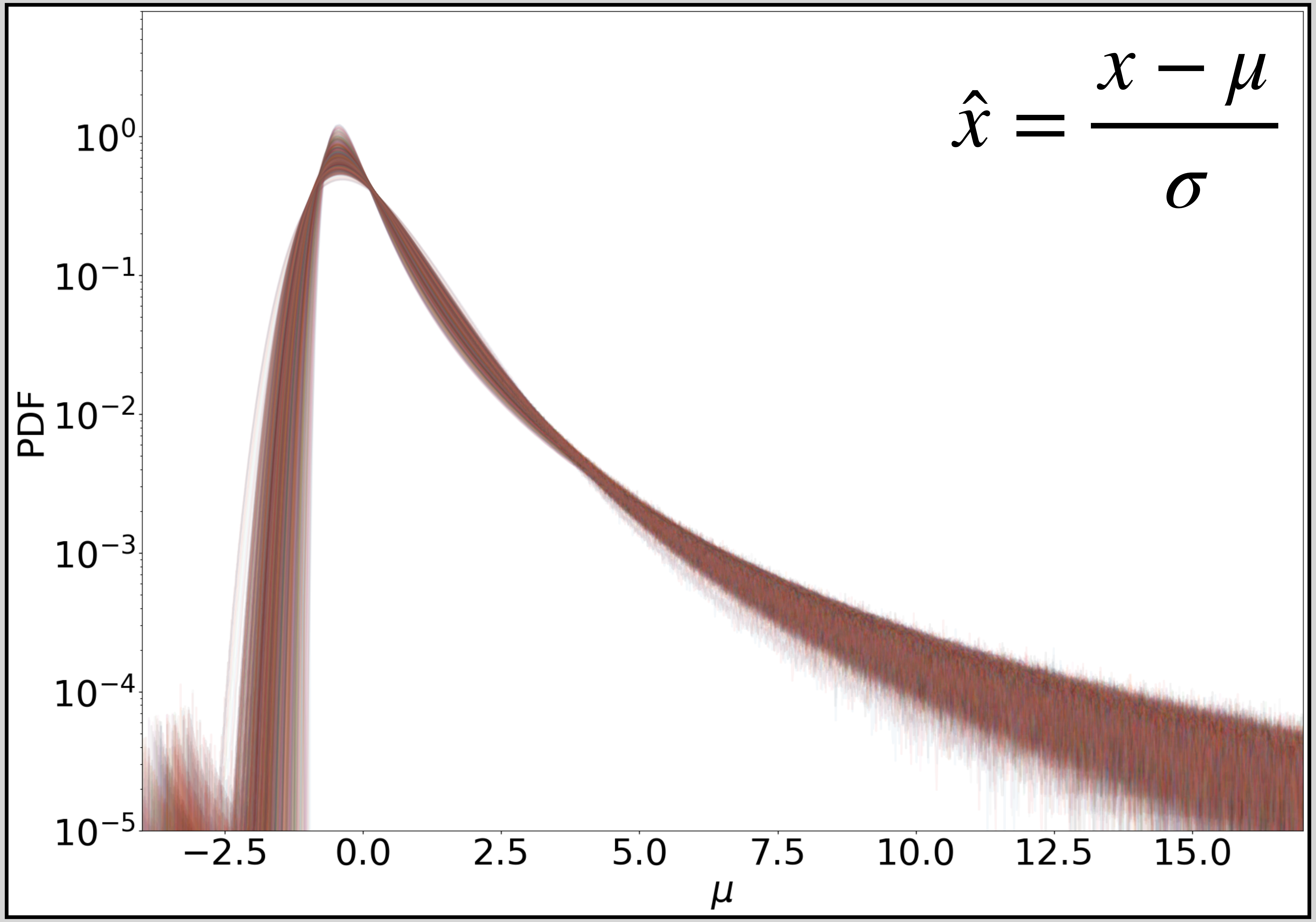

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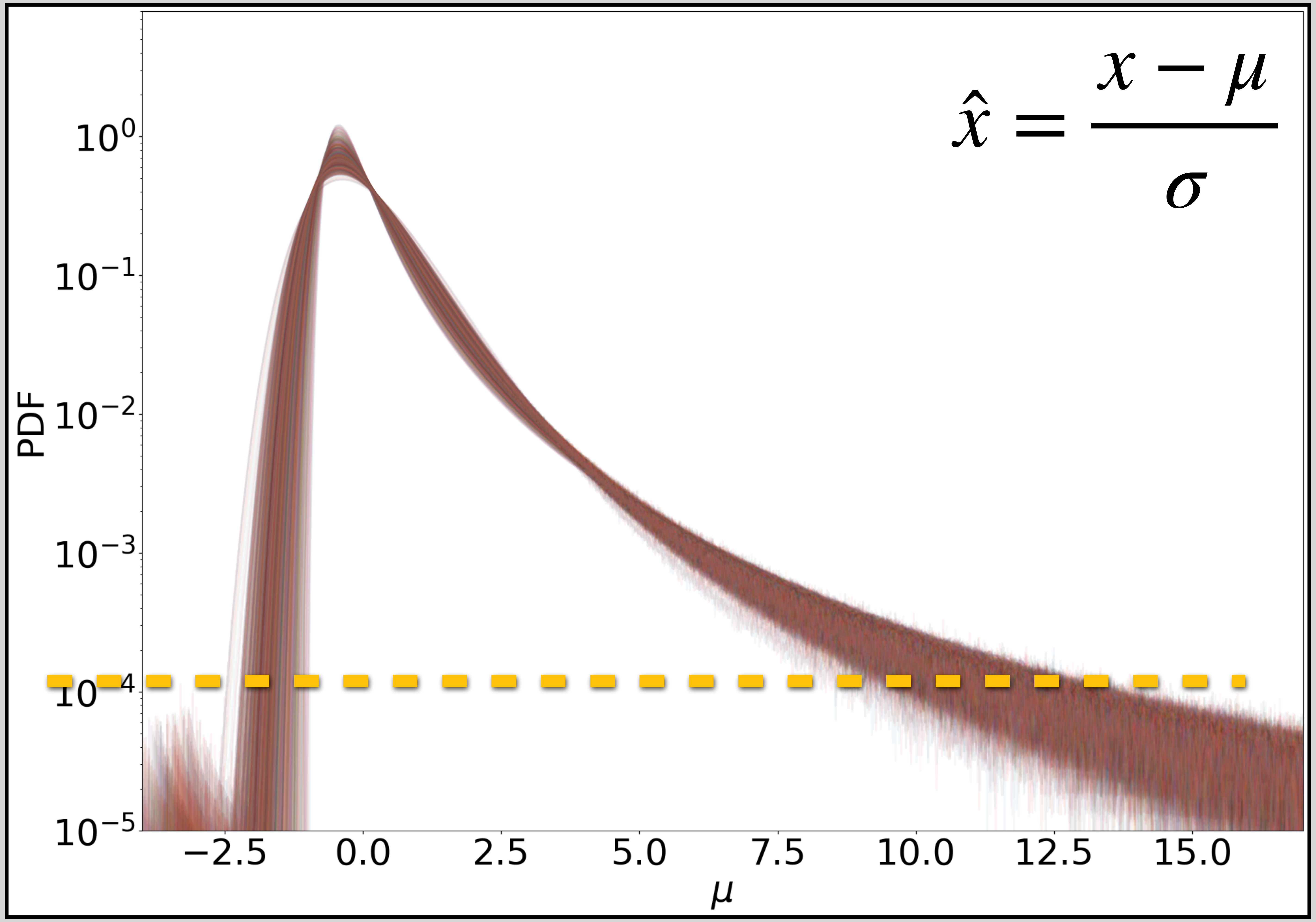

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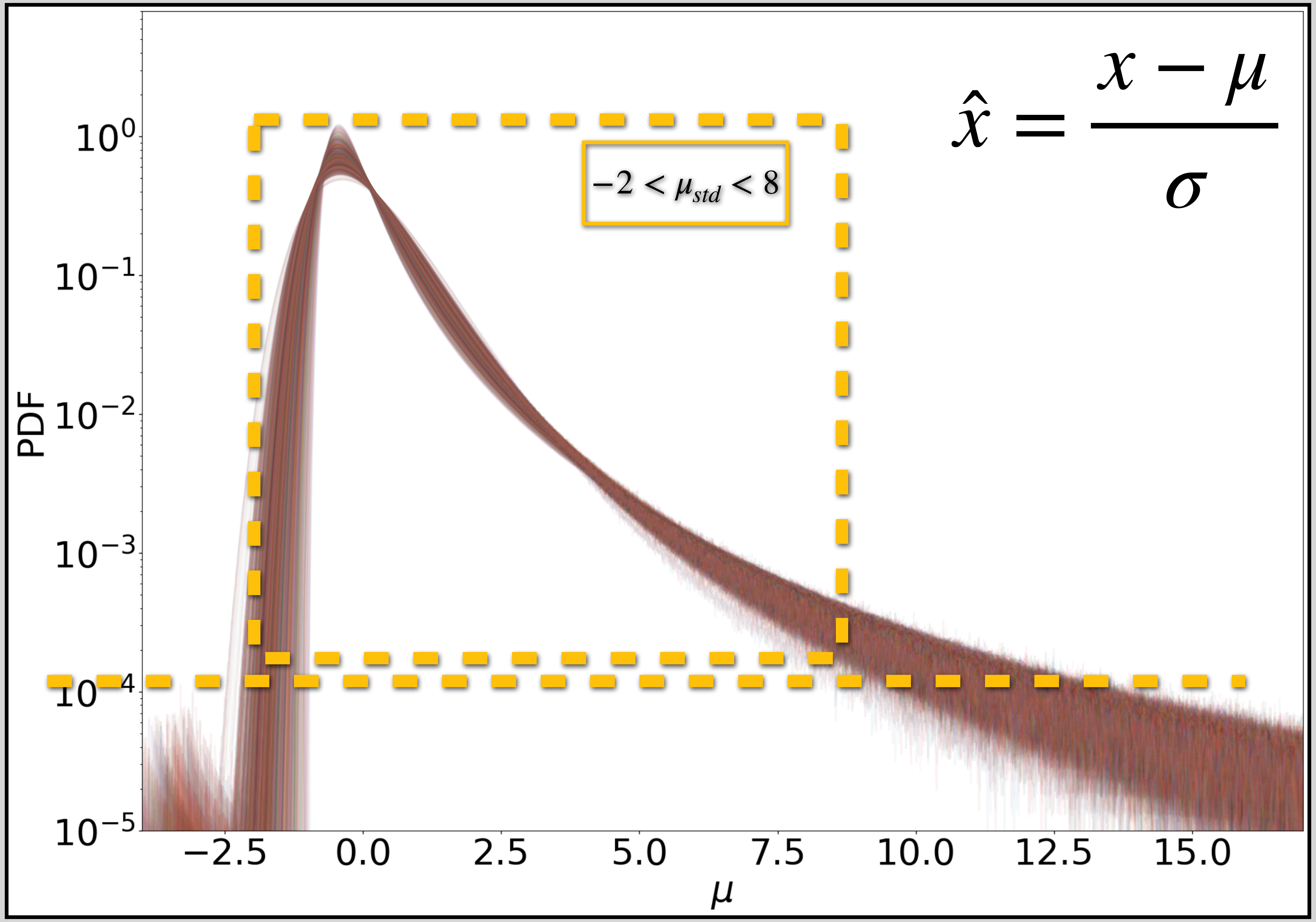

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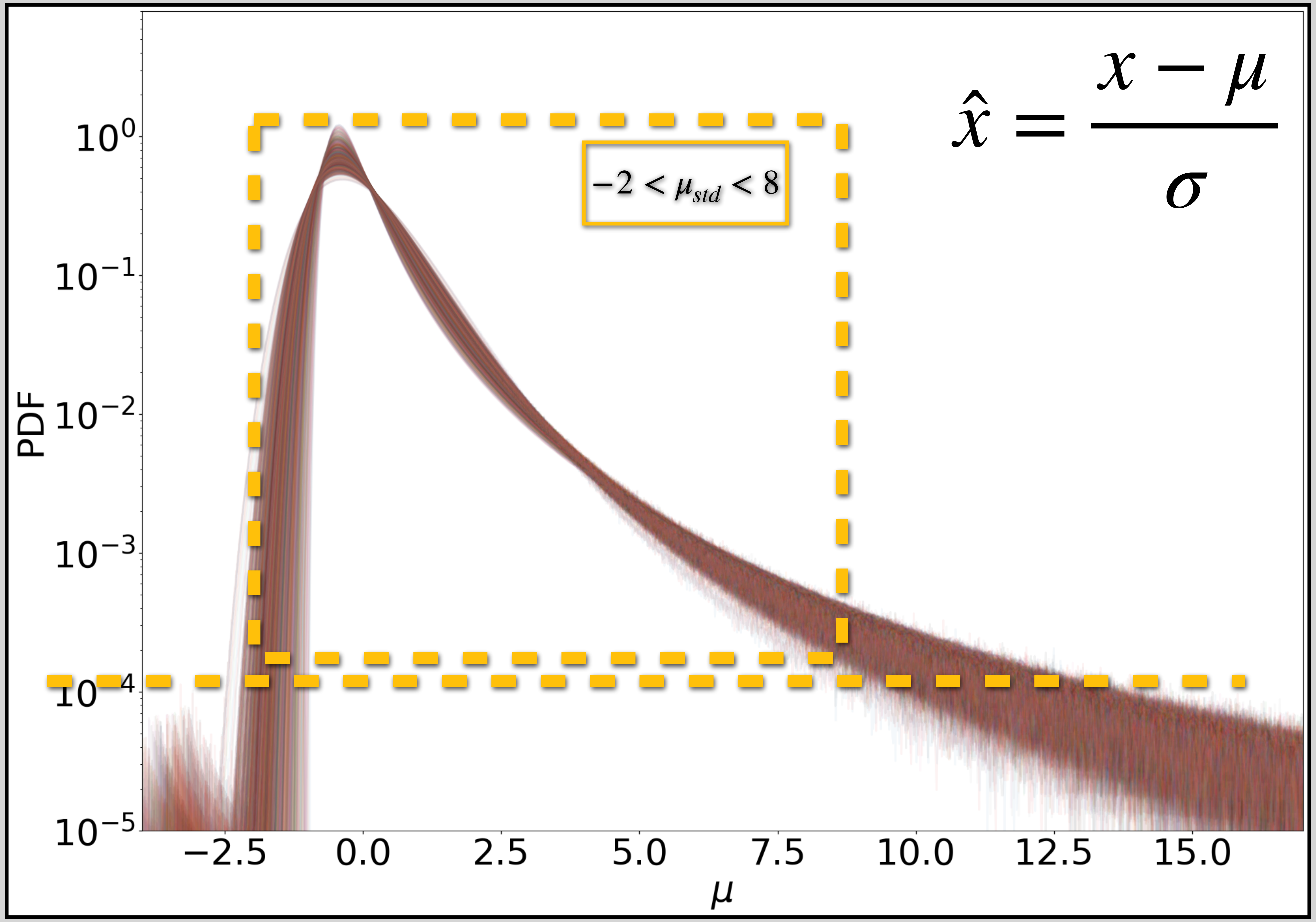

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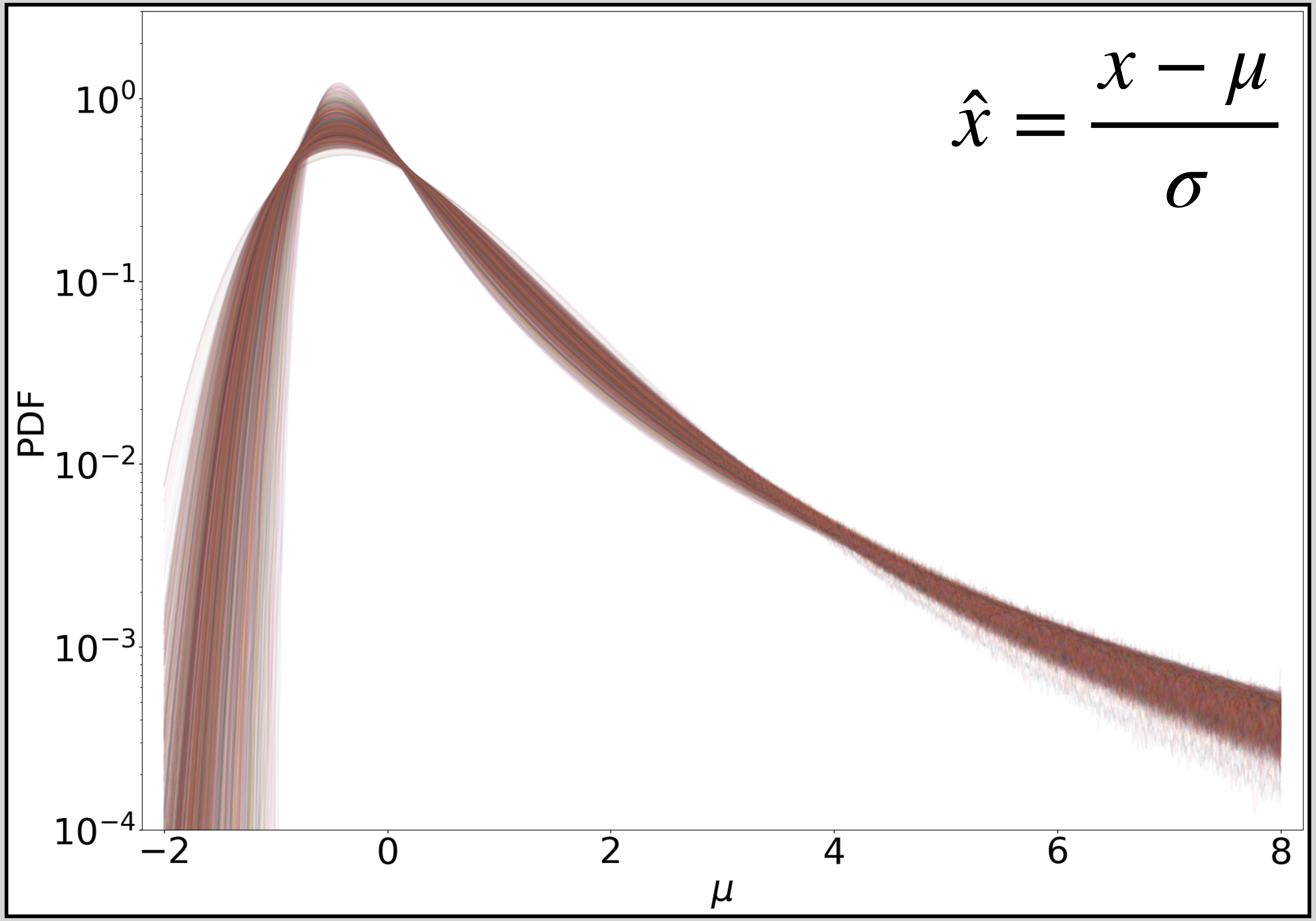
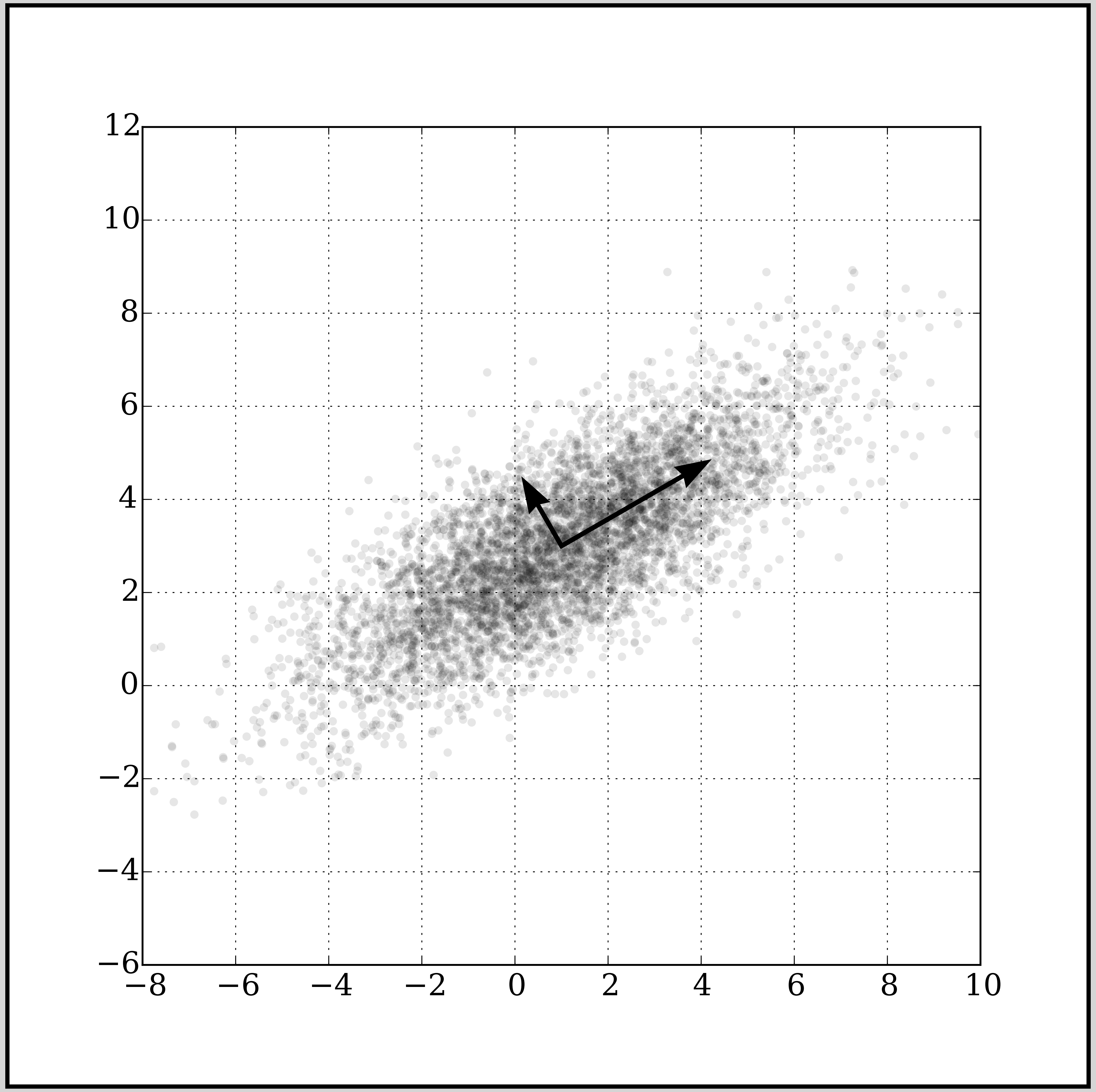


Principal Component Analysis



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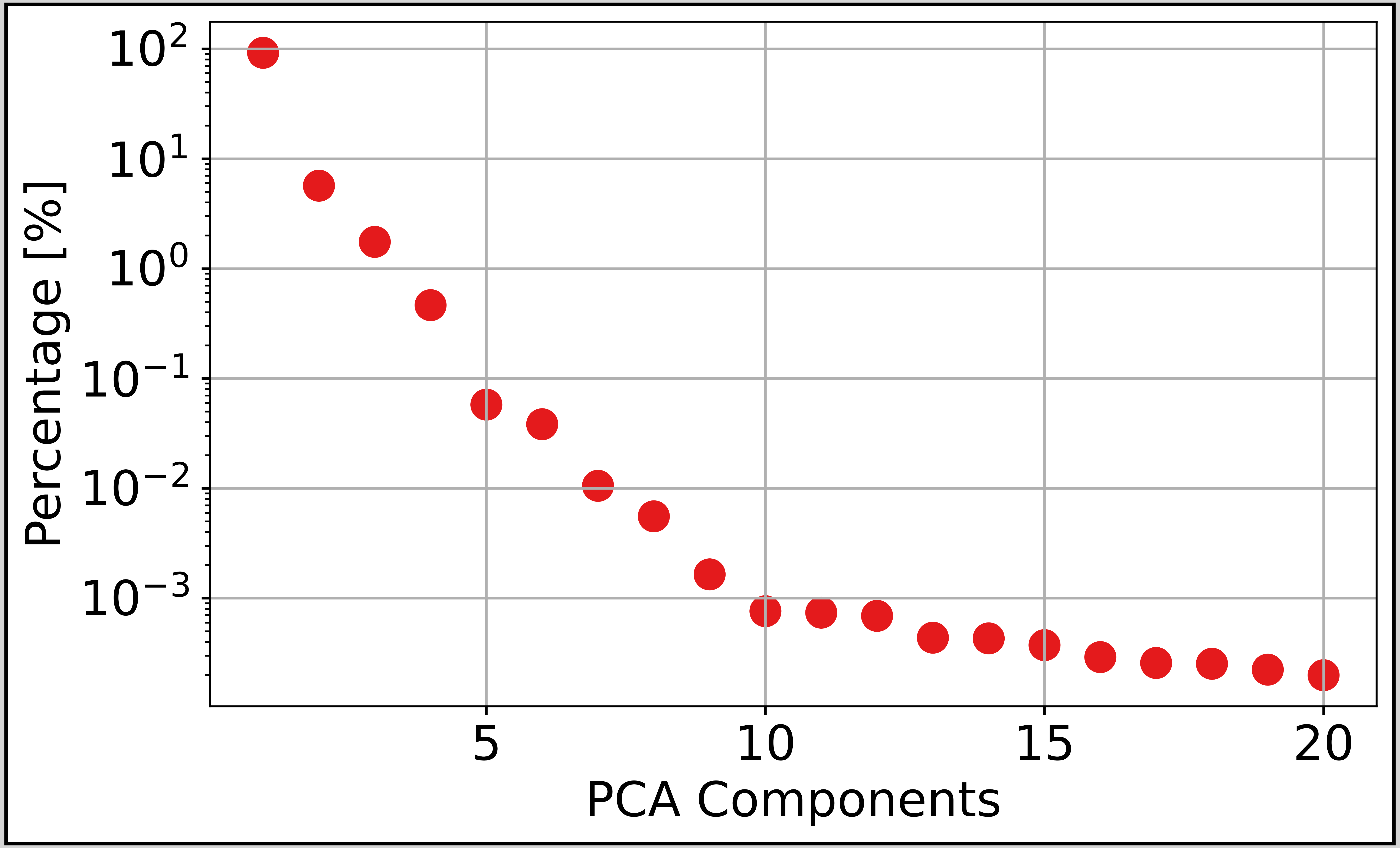
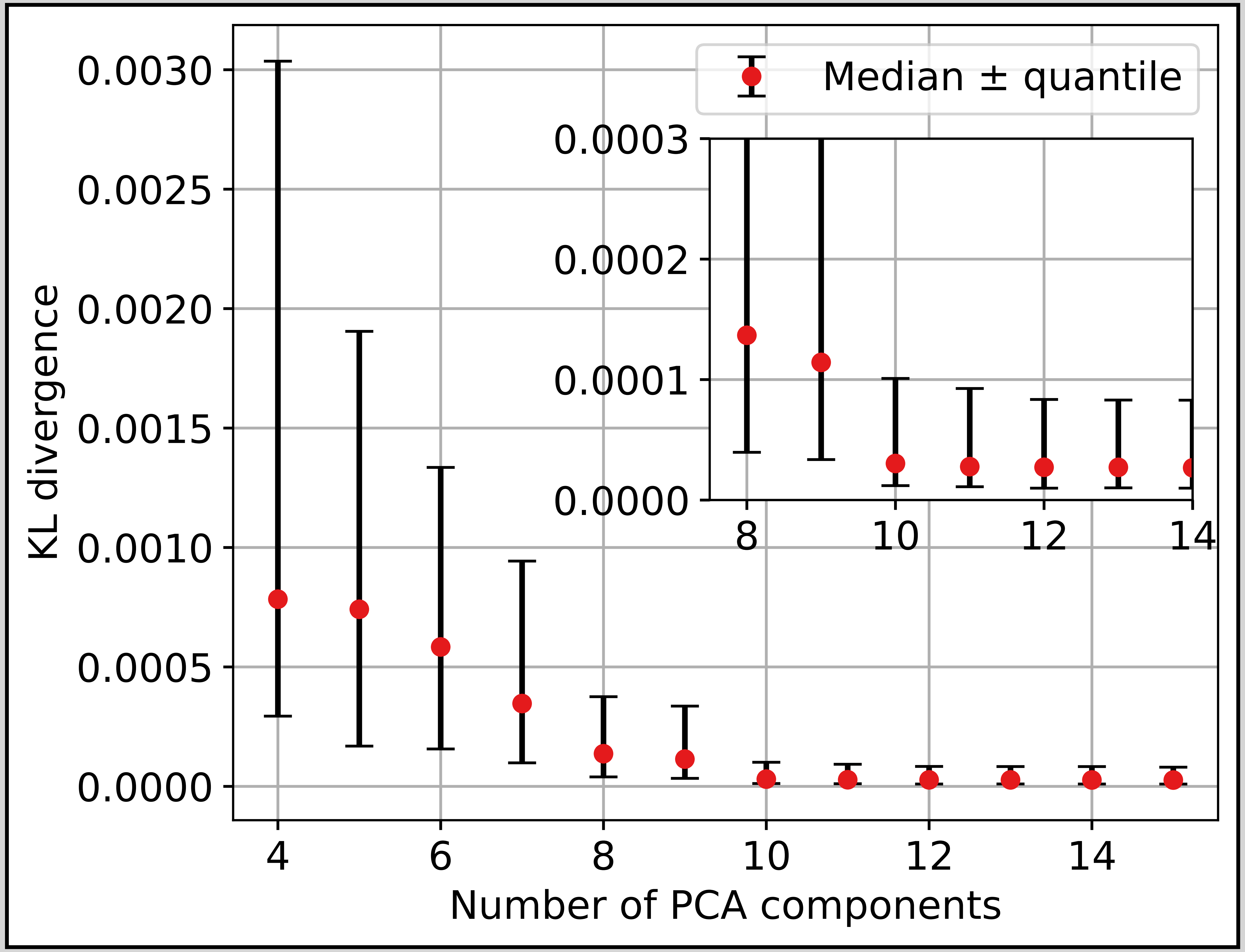
Principal Component Analysis



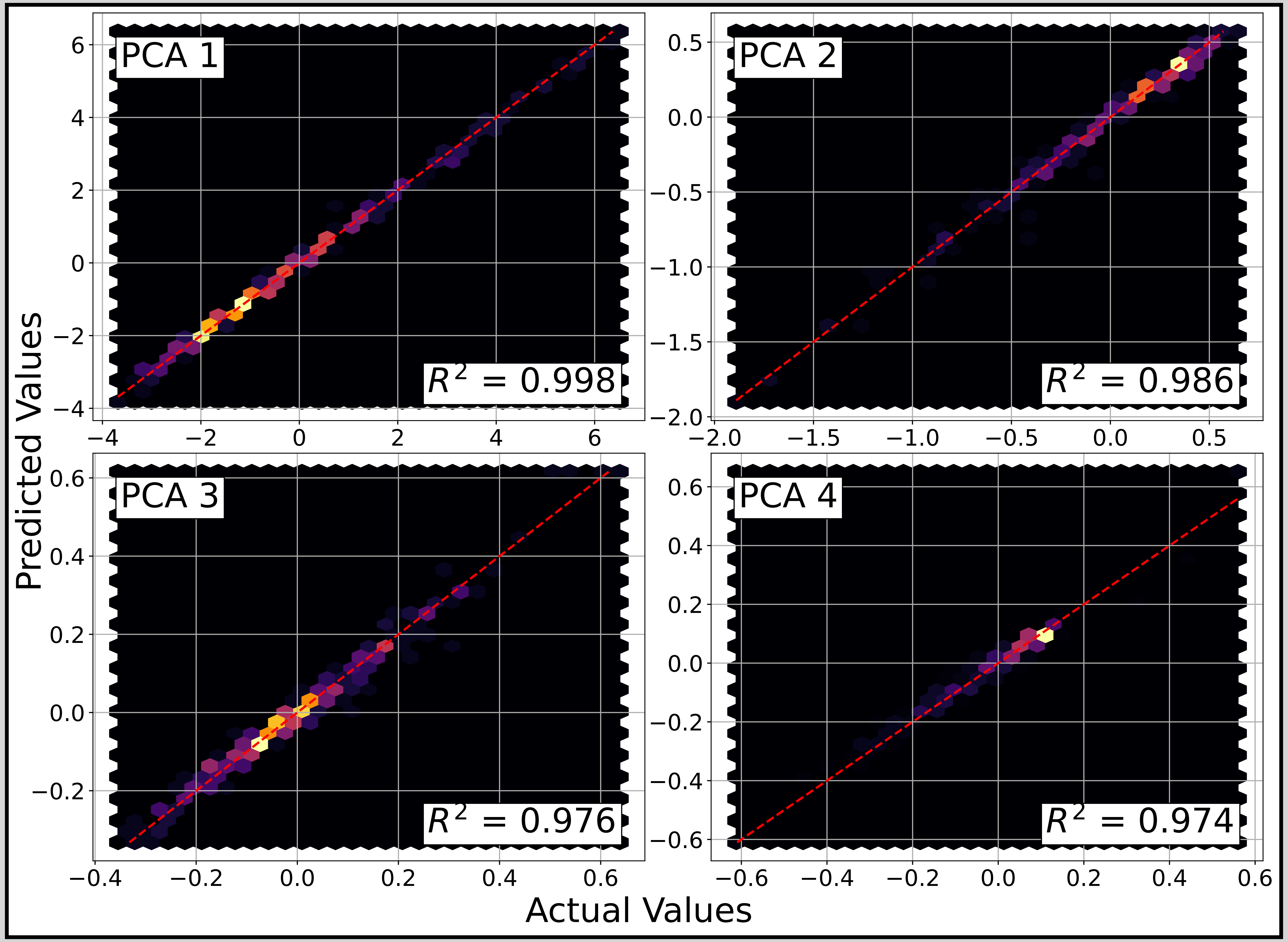
Principal Component Analysis

Kullback–Leibler (KL) divergence :

$$D_{\mathrm{KL}}(P \parallel Q) = \sum P(x) \log\left(\frac{P(x)}{Q(x)}\right)$$

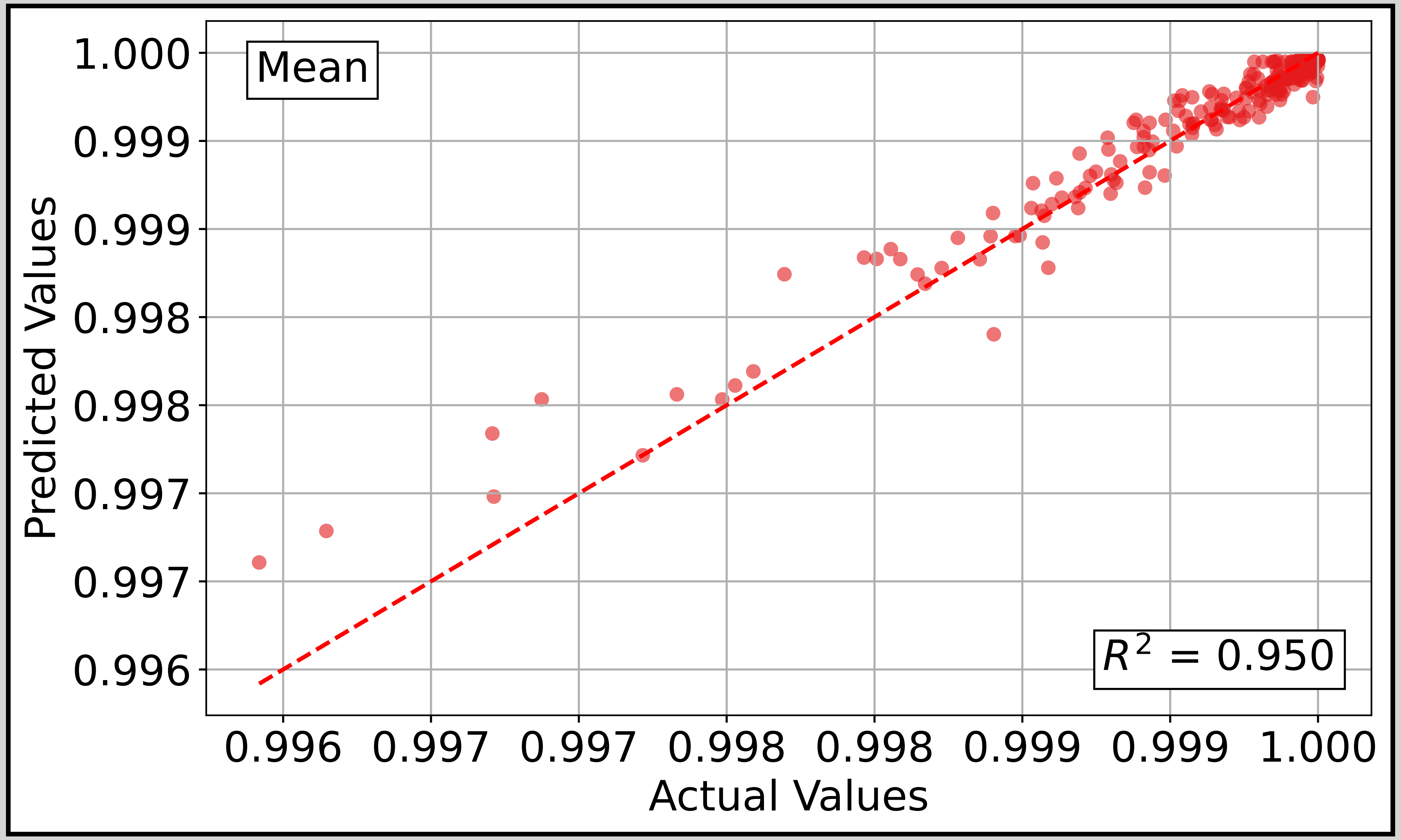


XGBoost

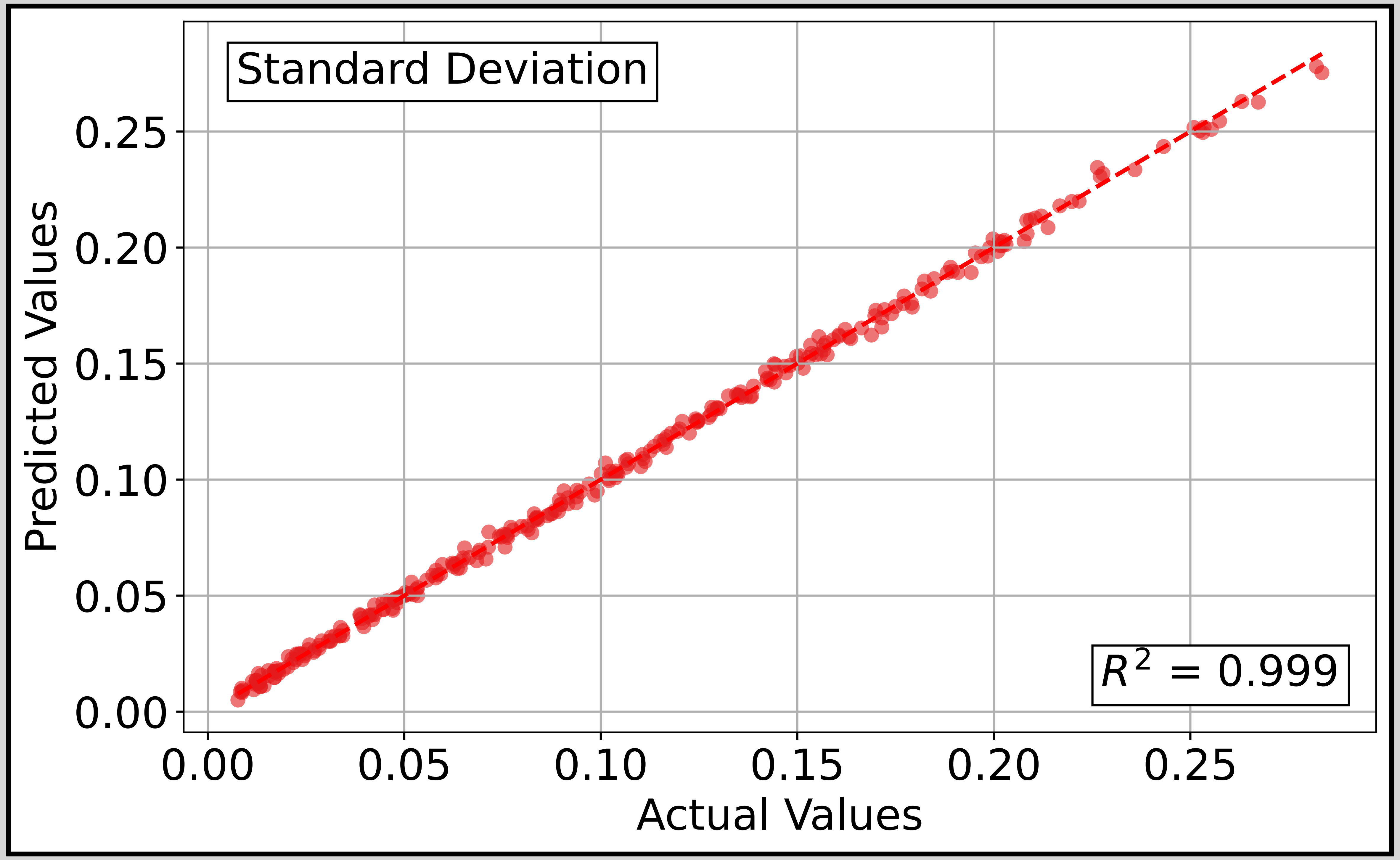


PCA 1
R^2 = 0.998
PCA 2
R^2 = 0.986
PCA 3
R^2 = 0.976
PCA 4
R^2 = 0.974
Predicted Values
Actual Values

XGBoost



Mean
1.000
0.999
0.999
0.998
0.998
0.997
0.997
0.996
Predicted Values
0.996
0.997
0.997
0.998
0.998
0.999
0.999
1.000
Actual Values
R^2 = 0.950



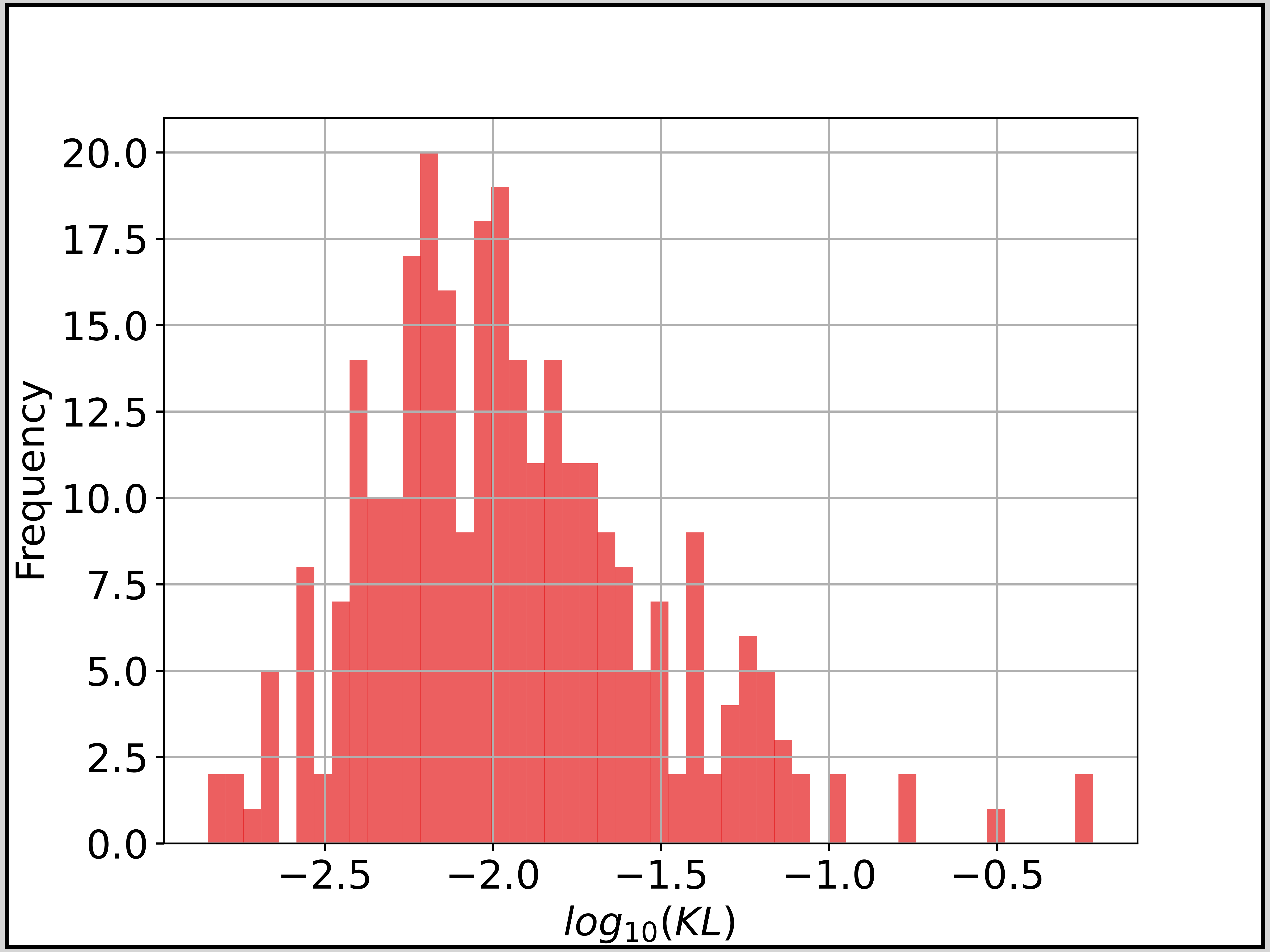
Standard Deviation
0.25
0.20
0.15
0.10
0.05
0.00
Predicted Values
0.00
0.05
0.10
0.15
0.20
0.25
Actual Values
R^2 = 0.999

Results

Mean KL Div.: 0.014

Median KL Div.: 0.007

Std. of KL Div.: 0.031



Results